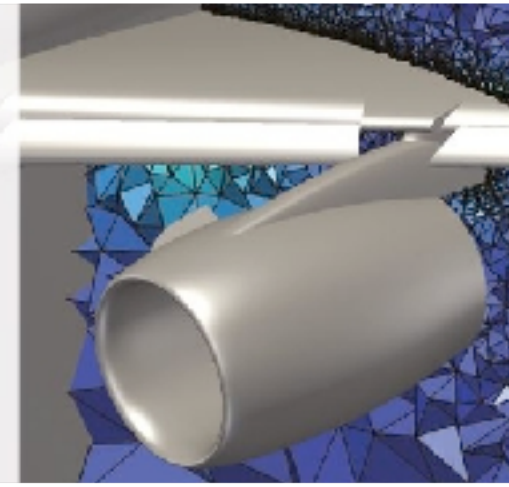


TETRAHEDRON VII

Workshop on Grid Generation for Numerical Computations
October 9-11, BSC-UPC, Barcelona, 2023



On CAD non-compliance in high-order curvilinear meshing

Kaloyan Kirilov, Jingtian Zhou and Joaquim Peiró
(Imperial College London)

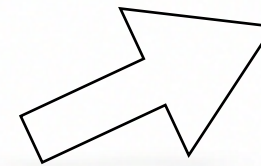
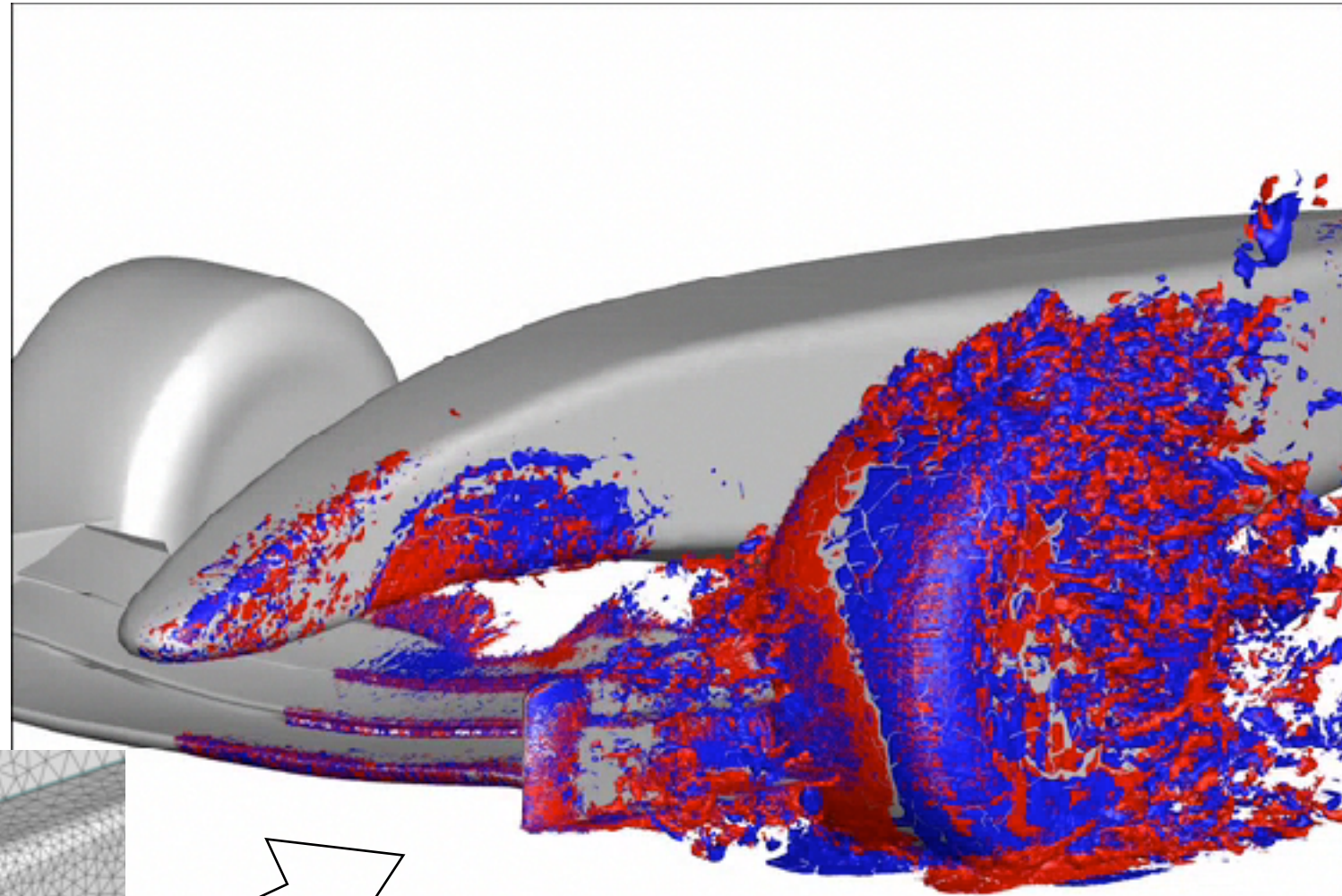
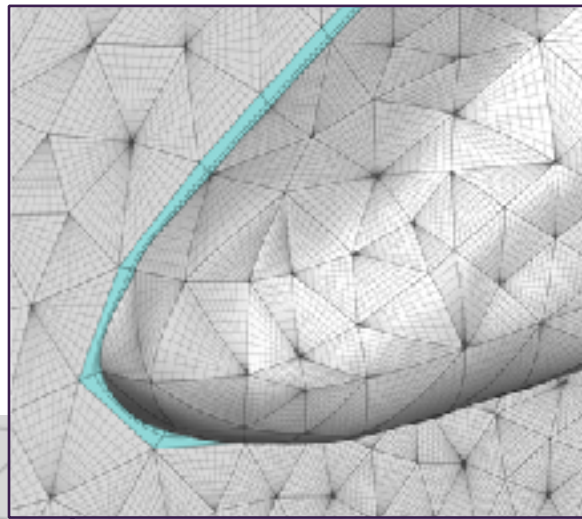
Mashy Green and David Moxey
(Kings College London)

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 955923.

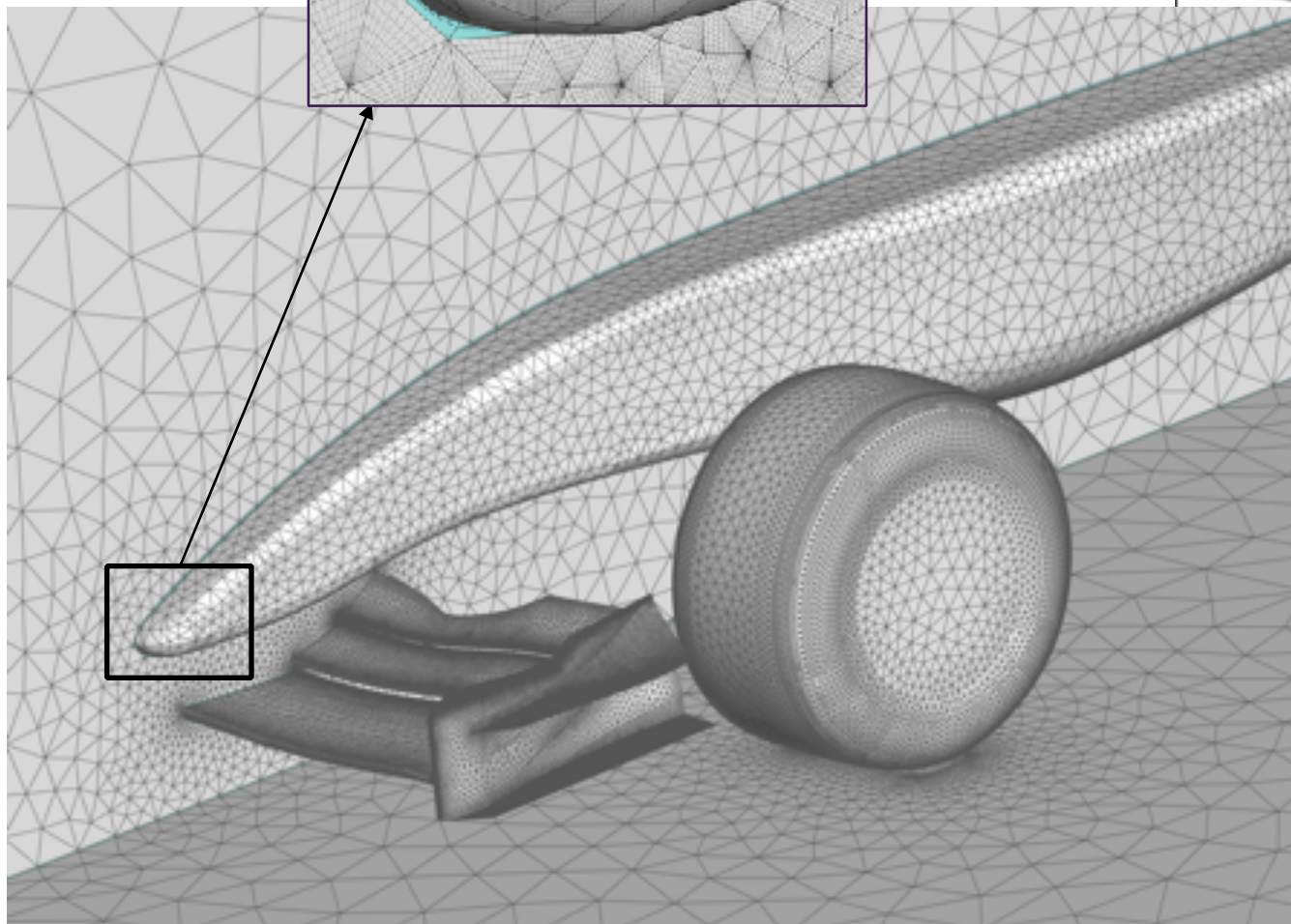


Complex geometries and flows

Mesh



Flow simulation



Meshes are curvilinear because ...

Complex geometries

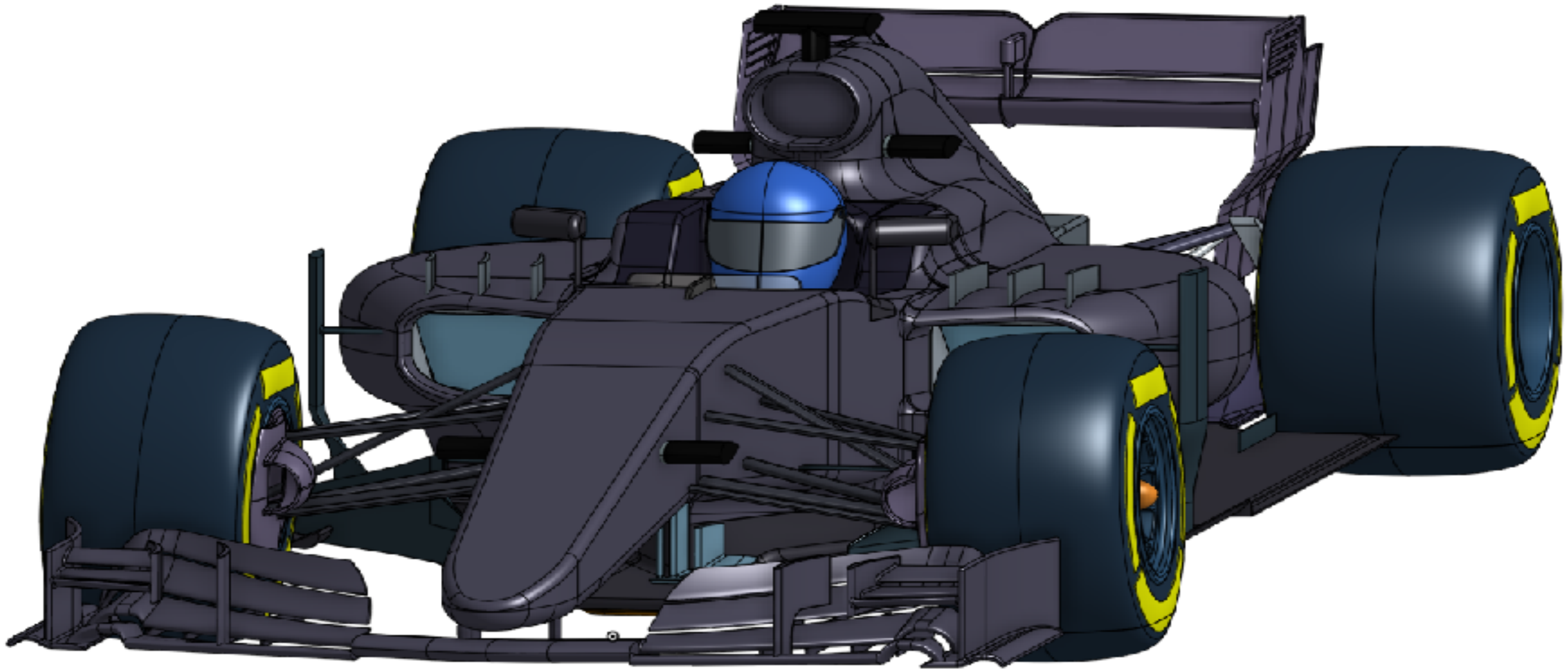


Look like this

Not like this

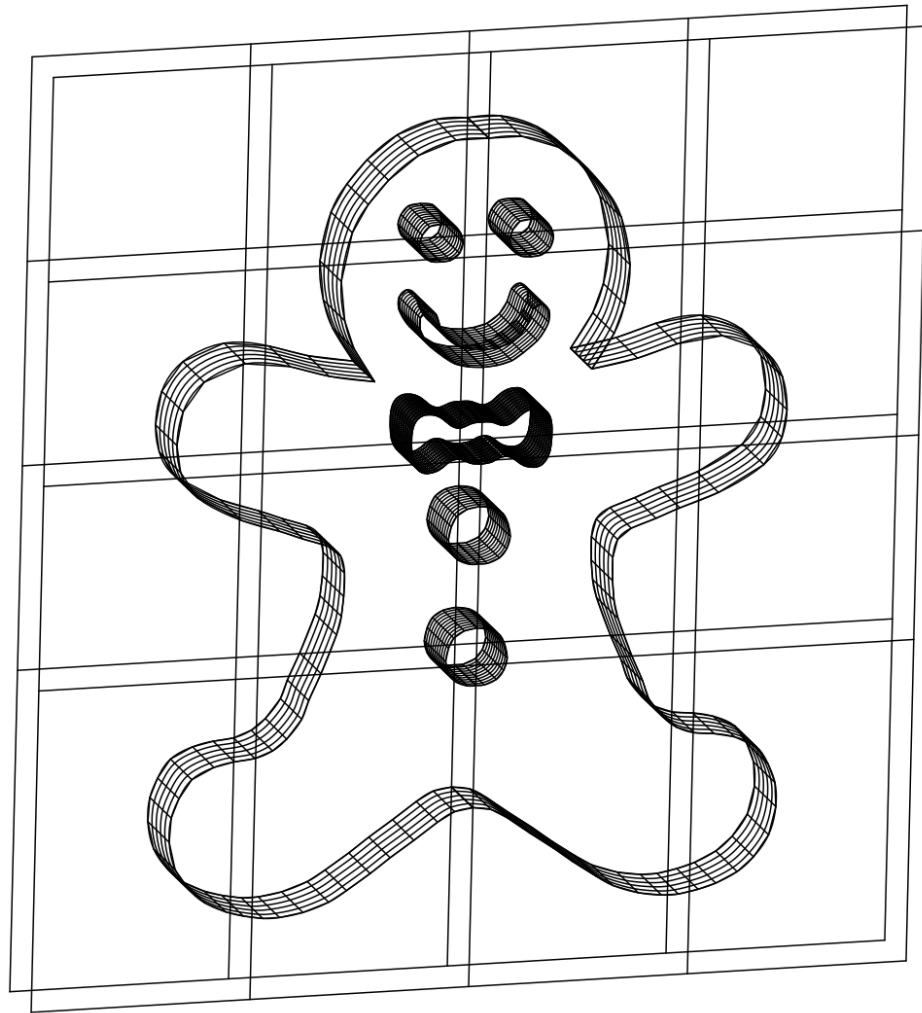


Target simulation

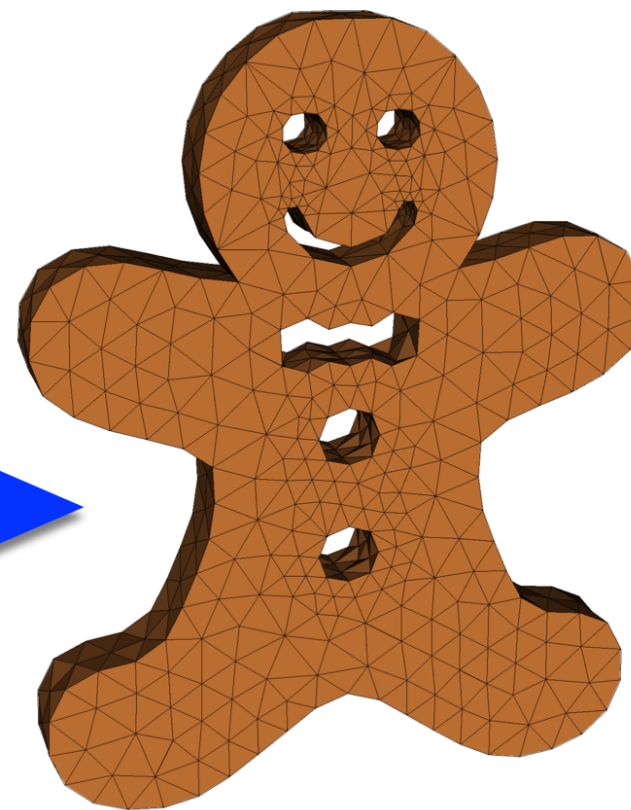


Perrinn F1 Open Geometry

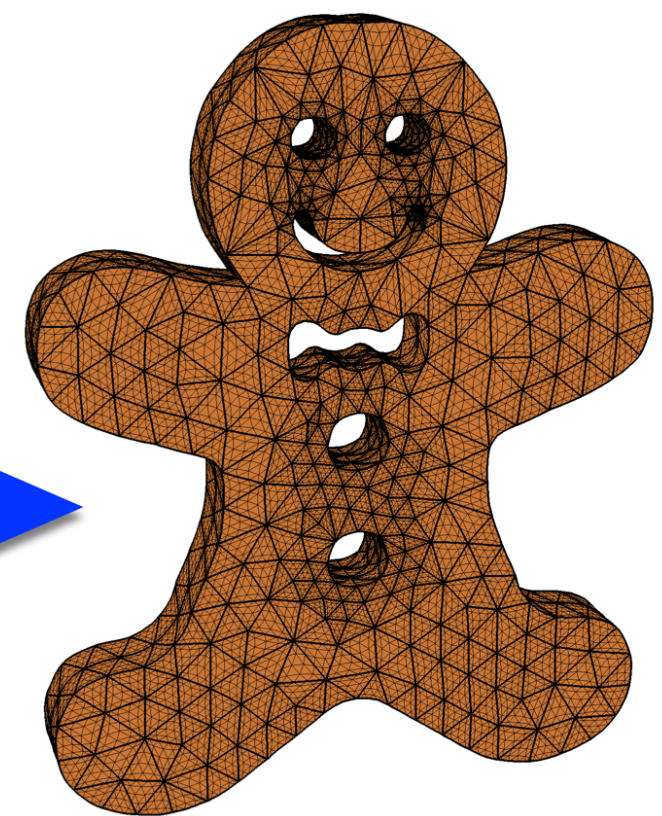
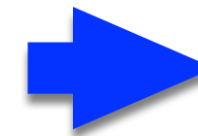
A posteriori high-order mesh generation



B-Rep



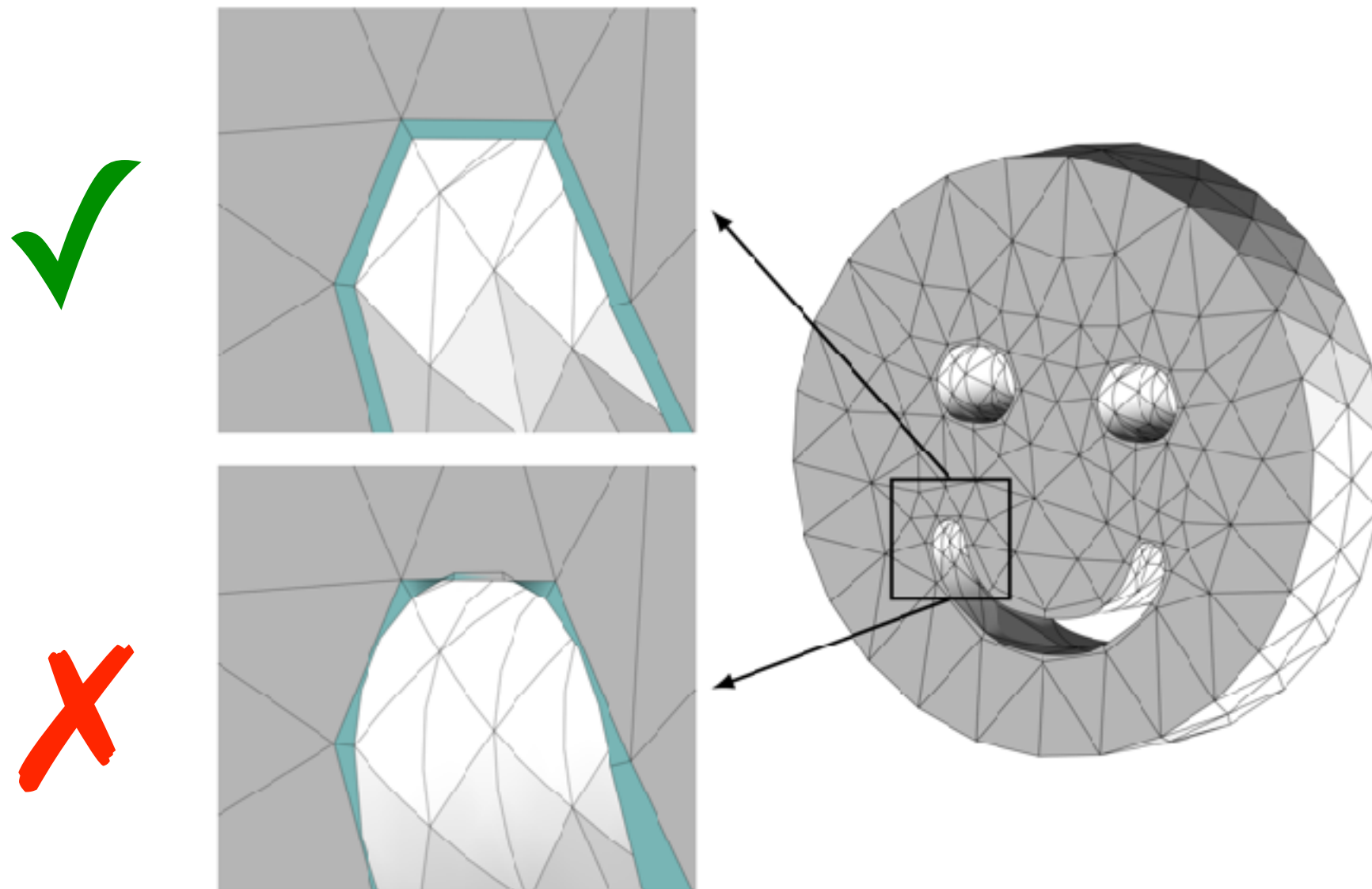
Linear mesh



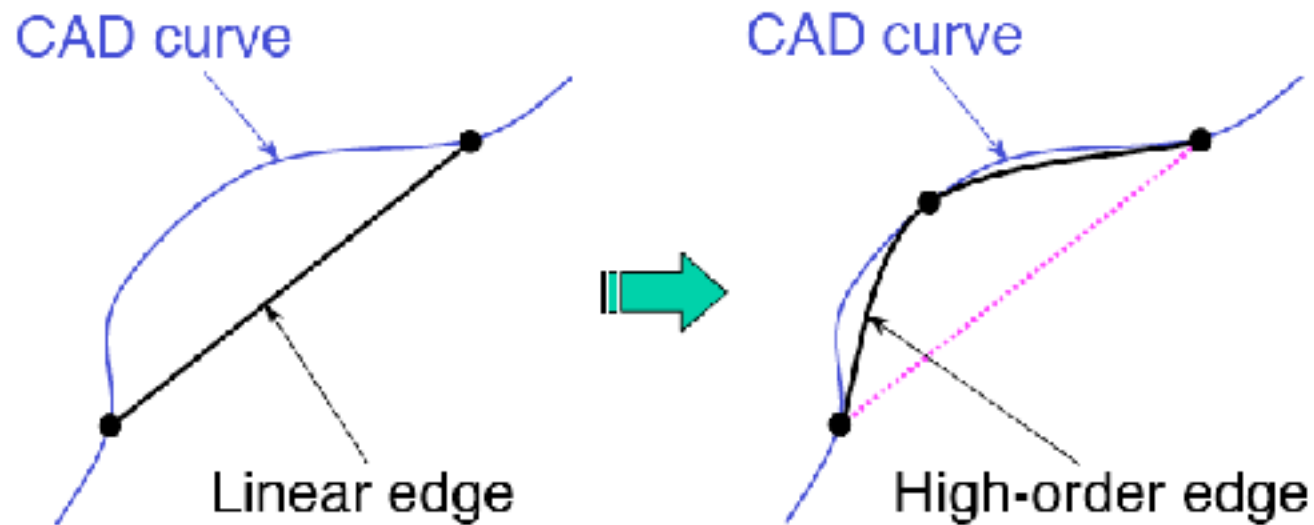
High-order mesh

A posteriori high-order mesh generation

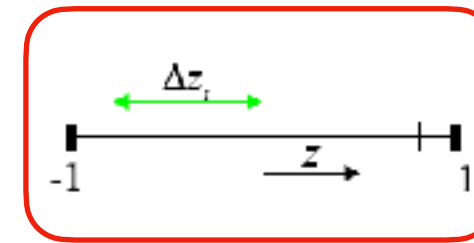
Curving mesh often leads to invalid elements



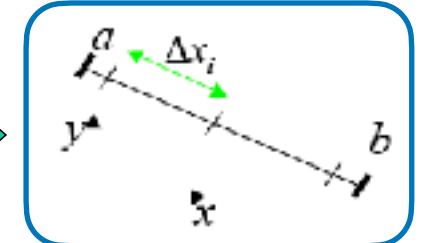
CAD's role in curving a linear mesh



Parametric space

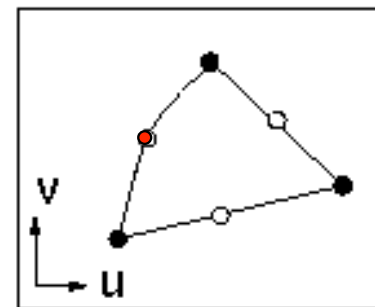
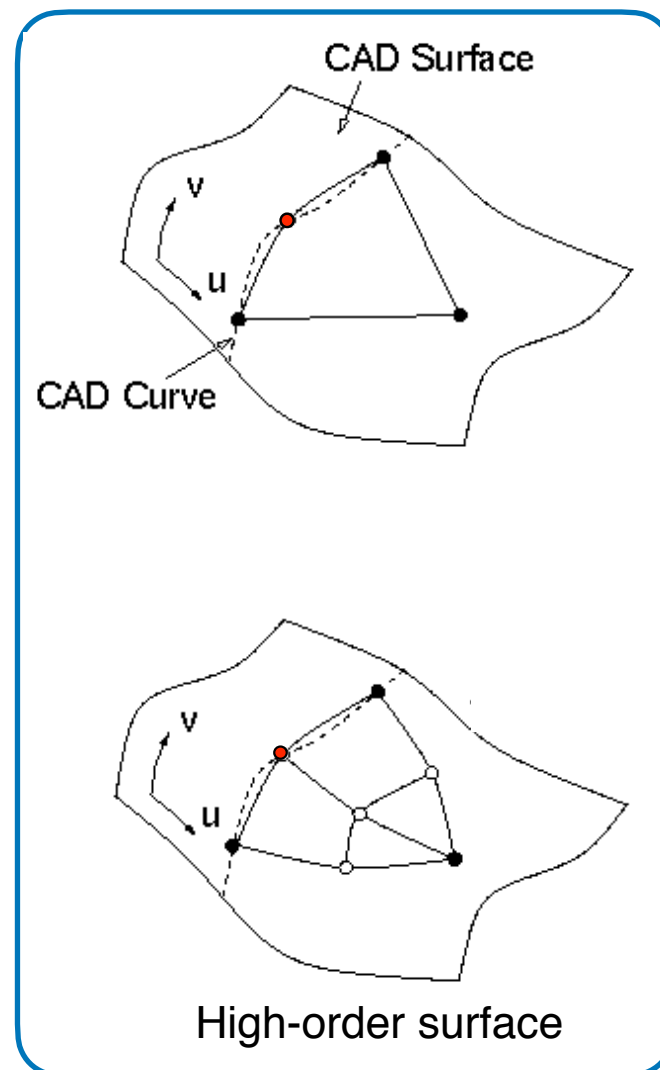


Physical space

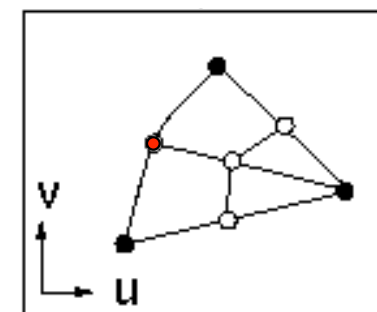


$$x_i = a \left(\frac{1 - \varepsilon_i}{2} \right) + b \left(\frac{1 + \varepsilon_i}{2} \right) \quad J = \begin{pmatrix} b - a \\ 2 \end{pmatrix}$$

Physical space



Internal edge split



Face approximation

Parametric space

CAD: The Missing Link

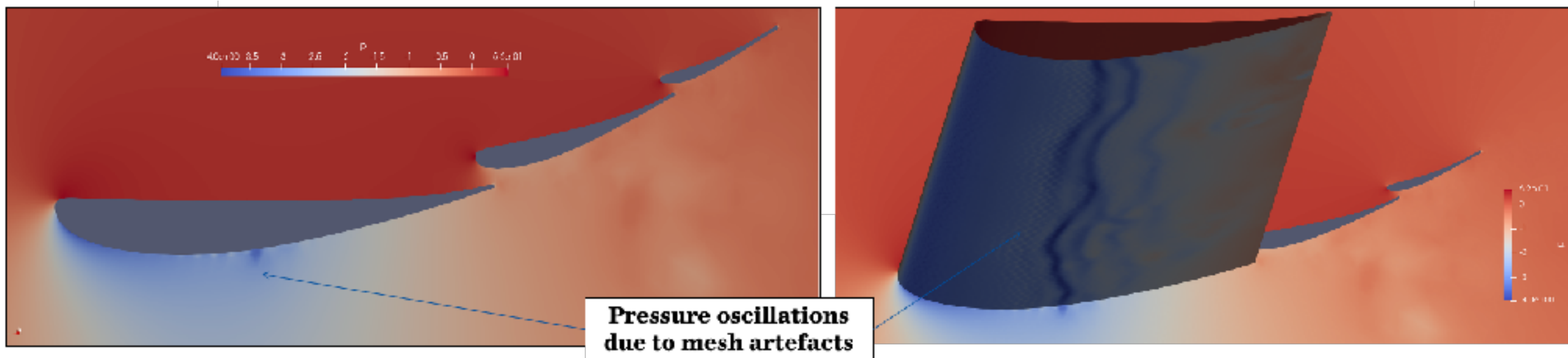
All generators use CAD information to generate a straight-sided mesh, but some do not provide this information to the user of the mesh.

This presents two issues:

1. Is the mesh boundary compliant?
2. How do we curve the mesh to be?

Effect of CAD non-compliance

Errors in boundary geometry could have a significant impact on simulation accuracy.

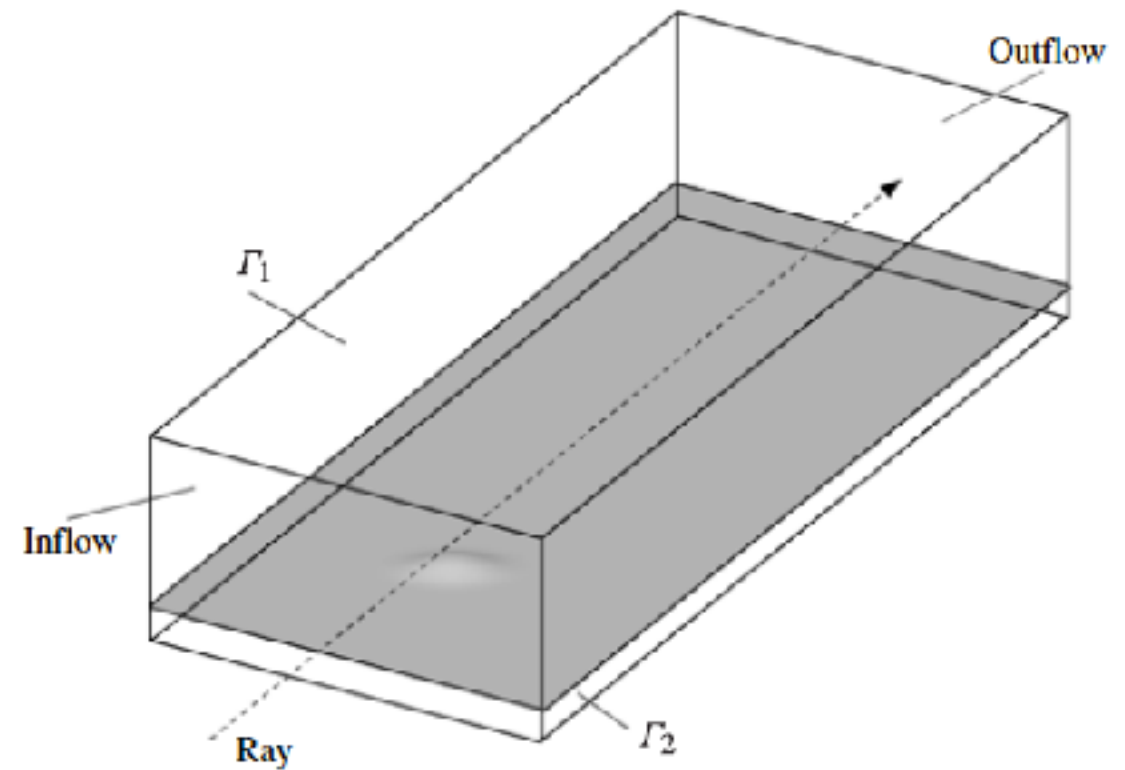
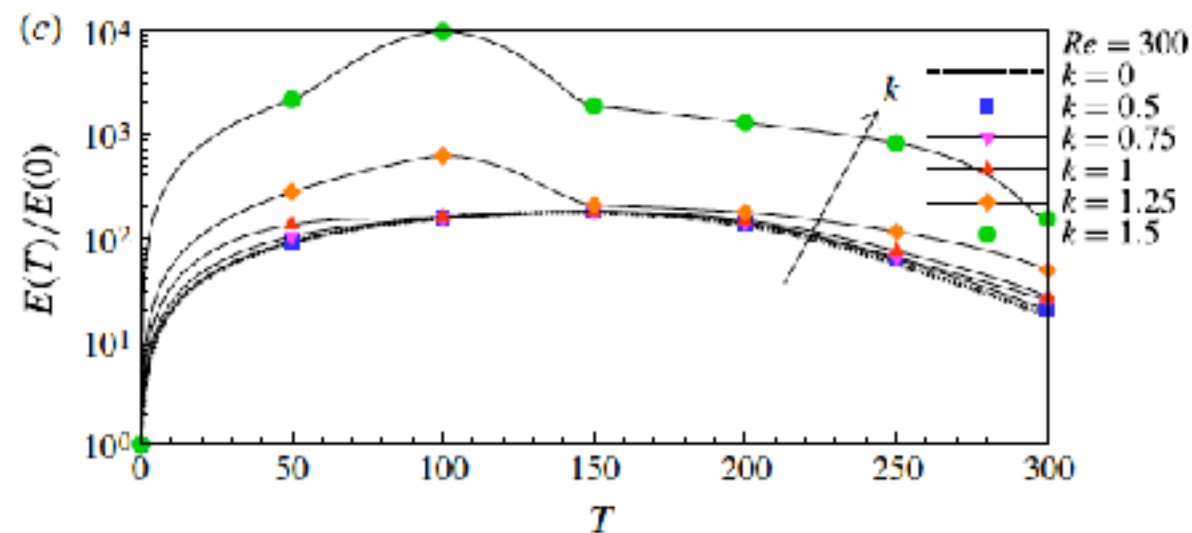
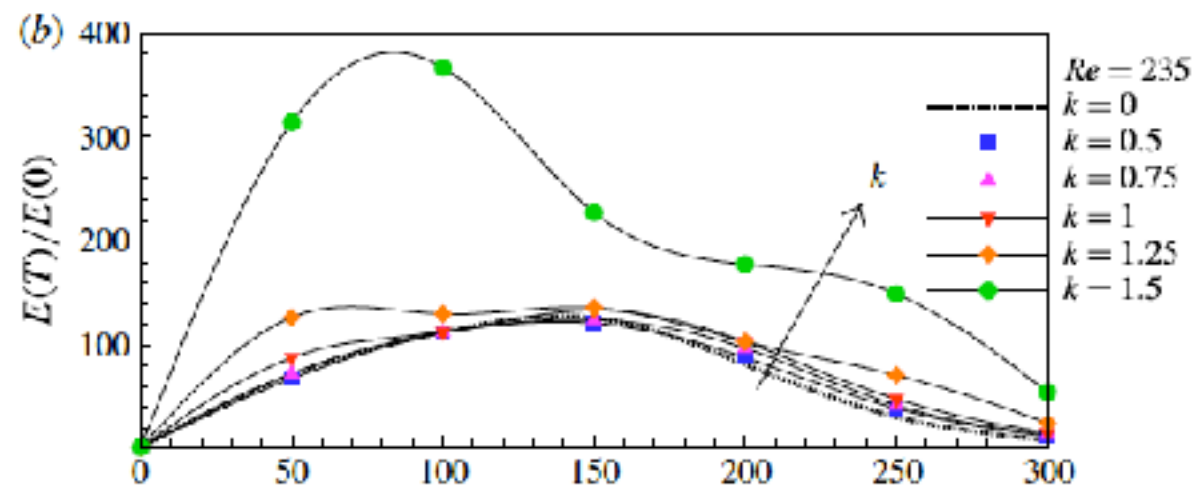
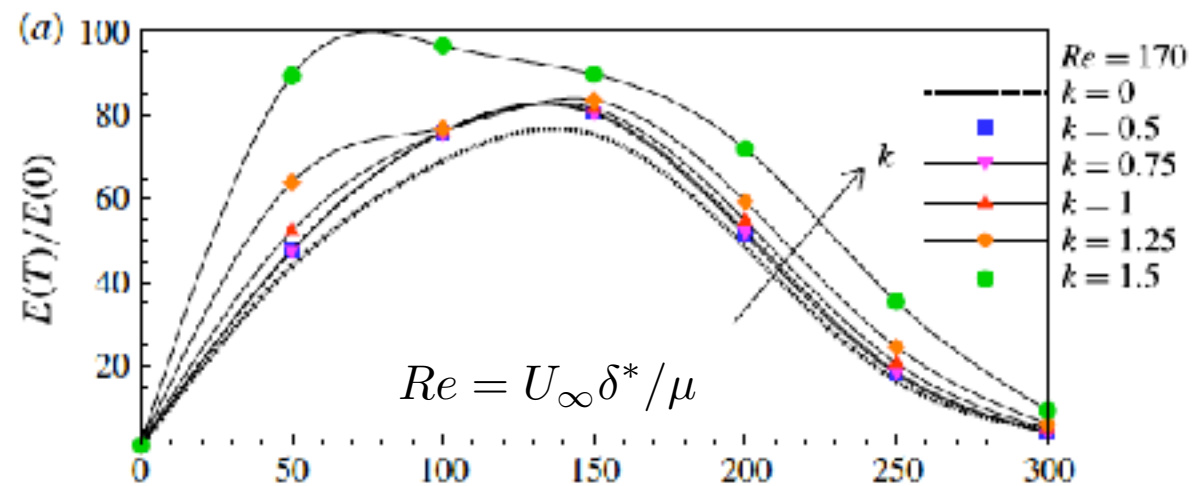


Small-scale roughness induces boundary-layer perturbations that destabilize the separated shear layer and lead to the development of turbulence downstream.

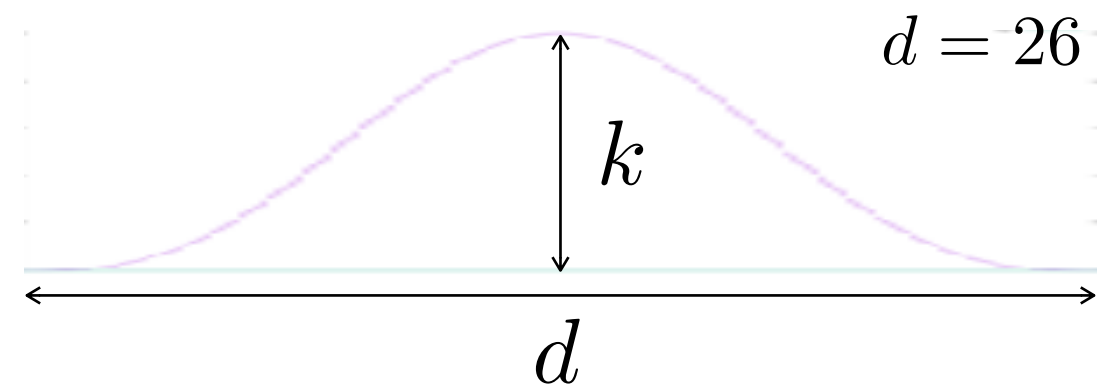
High-order methods are particularly susceptible because perturbations induced by small-scale geometric imperfections are not attenuated by numerical dissipation.

Sensitivity of flows to surface imperfections

Disturbance energy $E(t) = \int_V (u'^2(t) + v'^2(t) + w'^2(t)) dV$



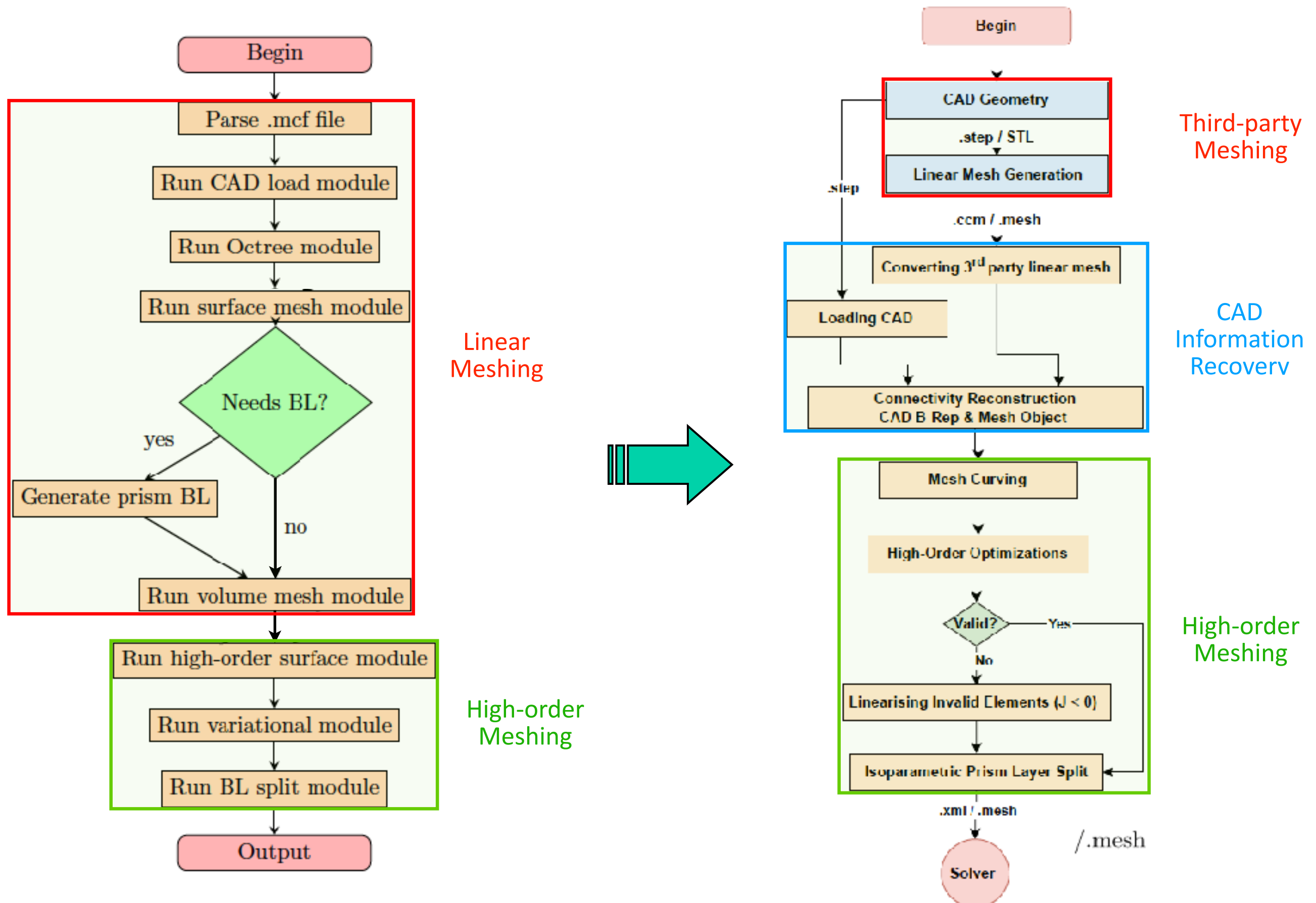
$$L_x = 216, L_y = 27.5, L_z = 72$$



$$h(r) = k \cos^3(\pi r/d)$$

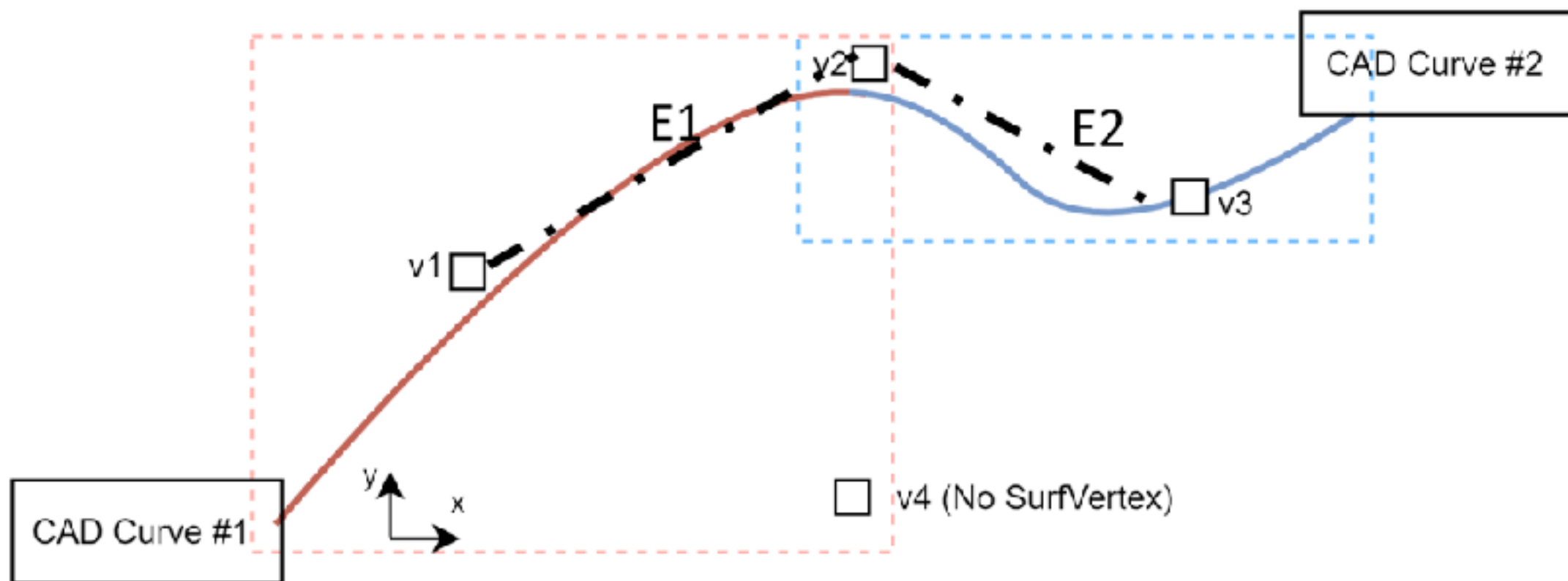
Transient growth in the flow past a three-dimensional smooth roughness element, S. Cherubini, M. D. De Tullio, P. De Palma and G. Pascazio, J. Fluid Mech. (2013), vol. 724, pp. 642–670.

Third-party mesh curving redesign



Recovering the CAD curve information

- Sample coordinates of CAD objects, calculate bounding boxes and store them in a k-D tree for efficient range search.
- Next steps:
 1. Associate vertex to CAD curve
 2. Project vertex onto curve



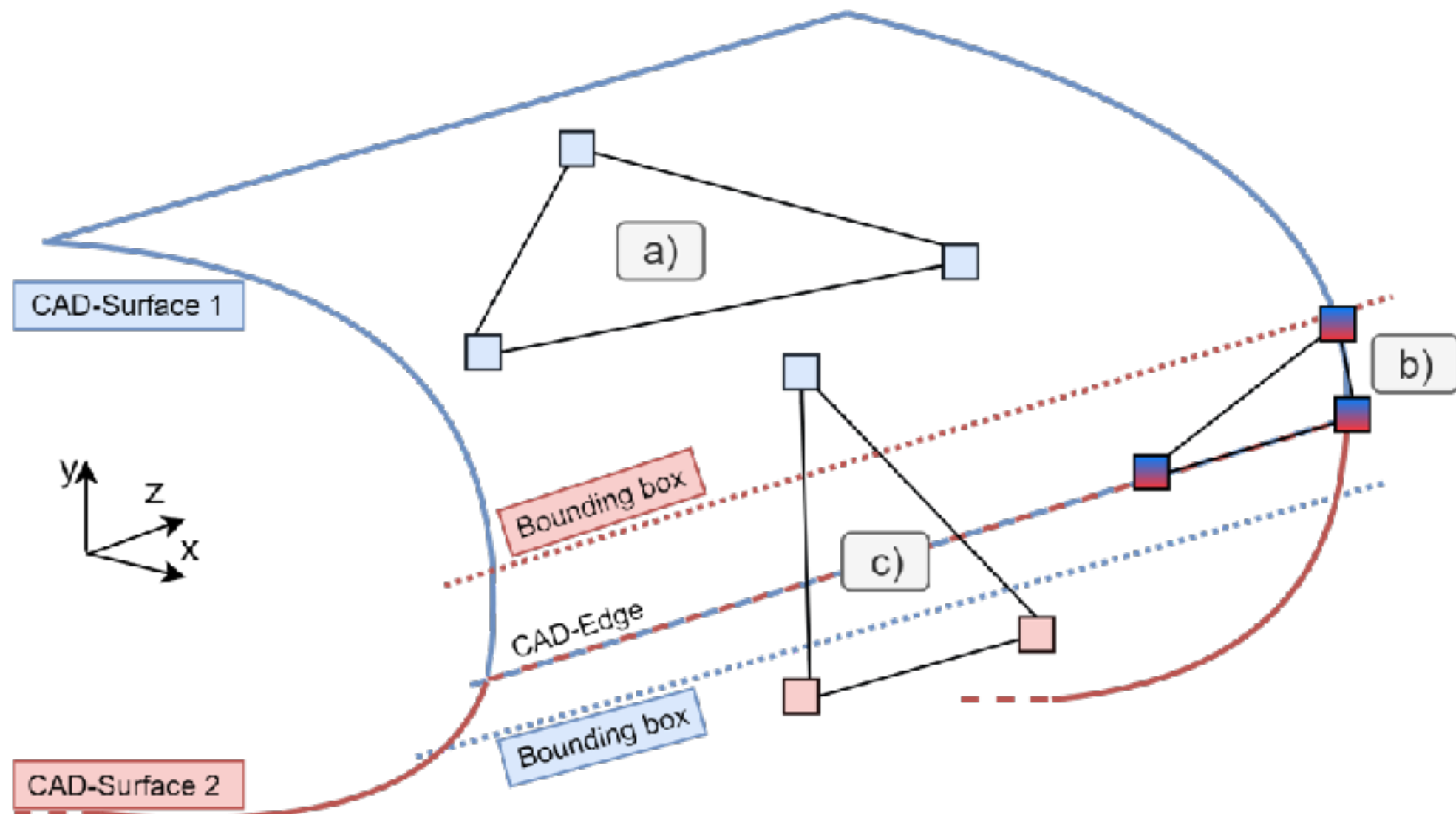
Note that none of the vertices is “exactly on” the CAD curve.

Candidate Curve ID?: v_1 to Curve #1; v_3 to Curve #2; v_2 to both curves

Recovering the CAD surface information

Possible cases:

- a) **Internal**: Element face on CAD surface
- b) **Corner**: Two face edges on CAD curves
- c) **Multi-patch**: Face spanning over two CAD surfaces

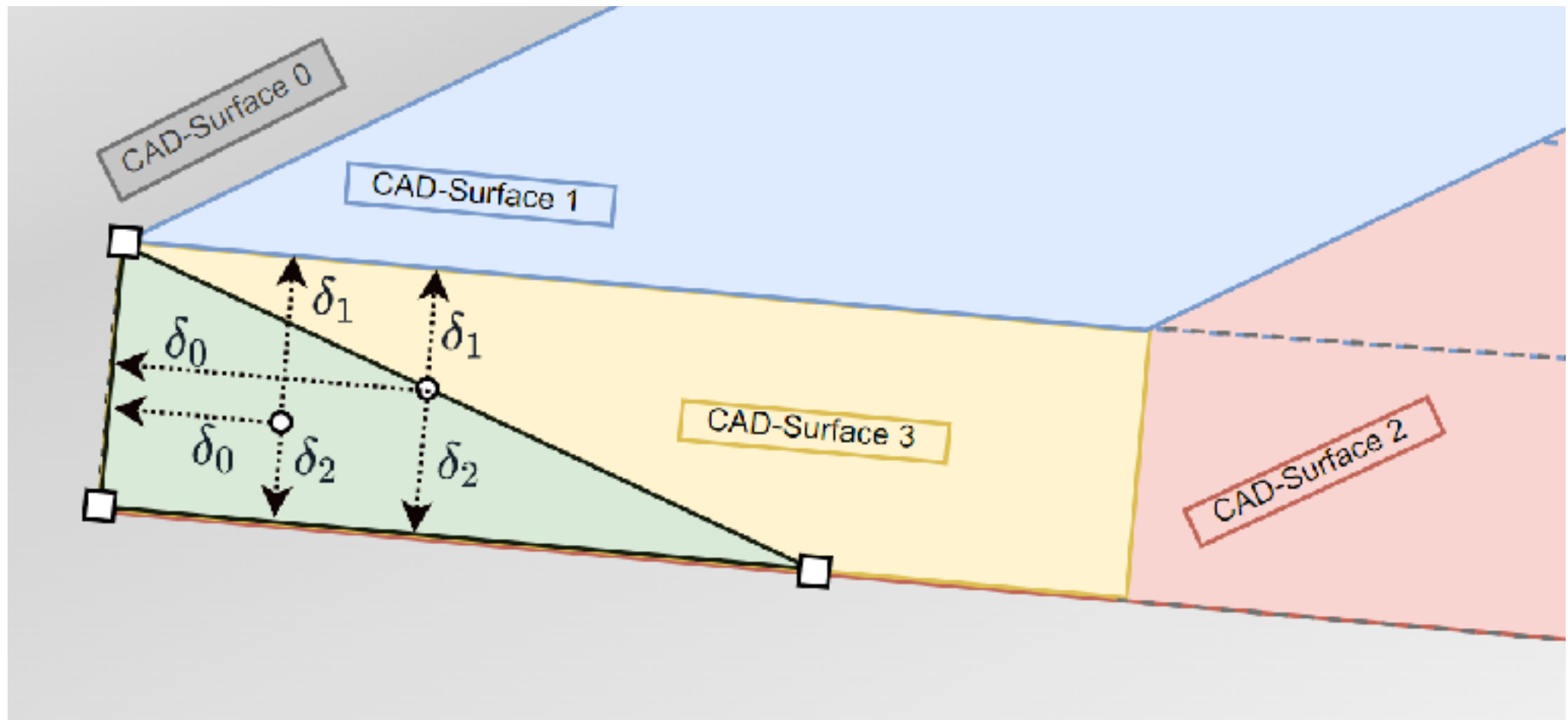


Multi-patch case

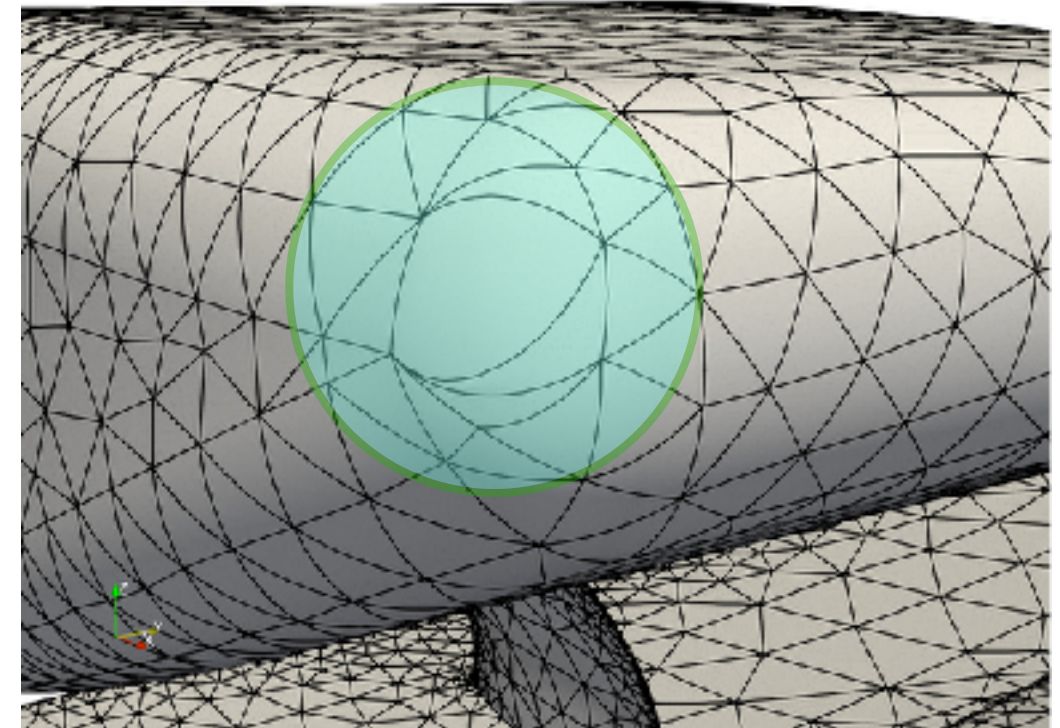
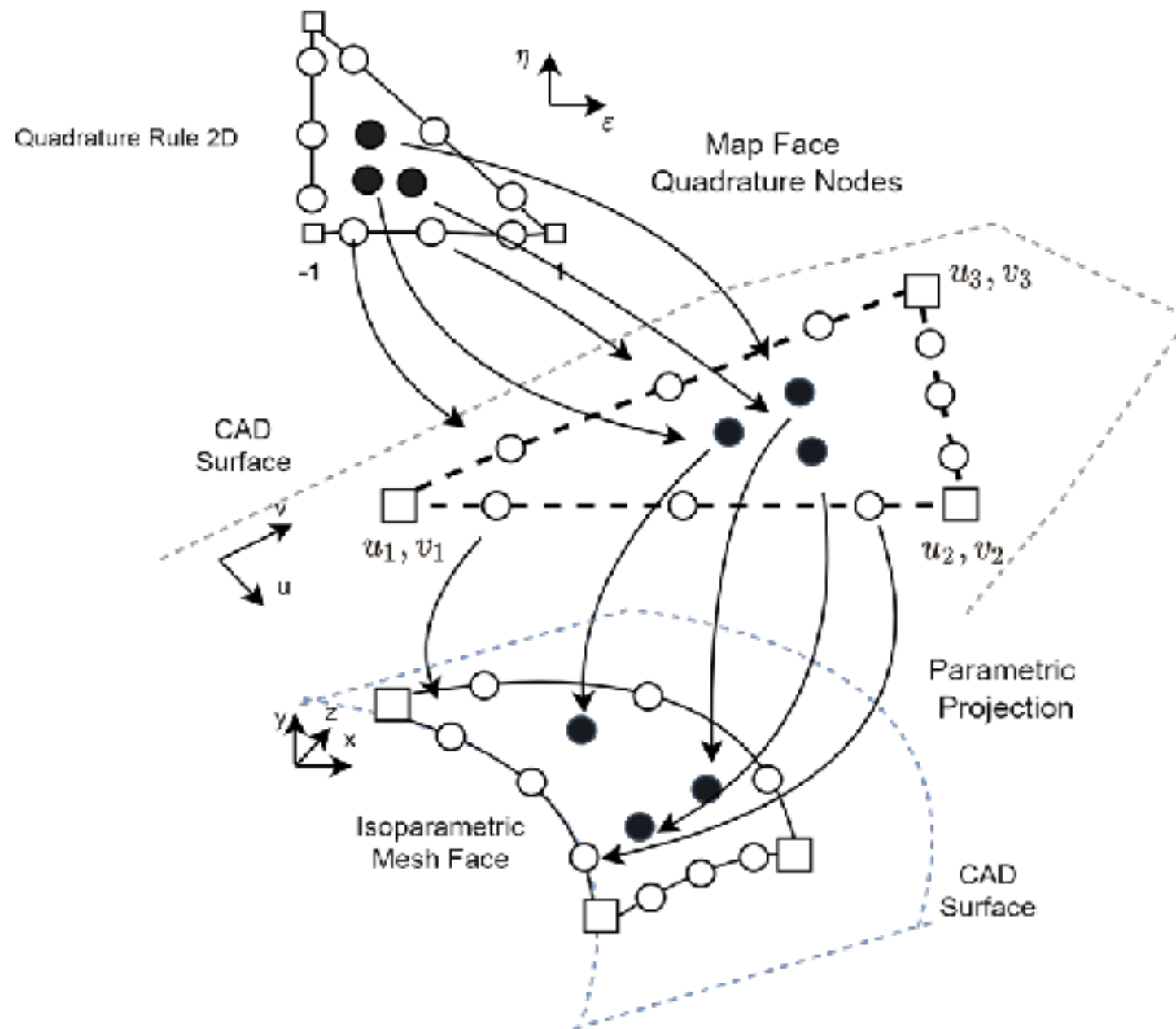
The surface ID of vertex $\mathbf{x}_j$ is obtained as

$$\text{ID} = \arg \min_k \sum_{j=1}^3 \left(\|\mathbf{x}_j - \mathbf{P}_k(\mathbf{x}_j)\| \right) \delta_k$$

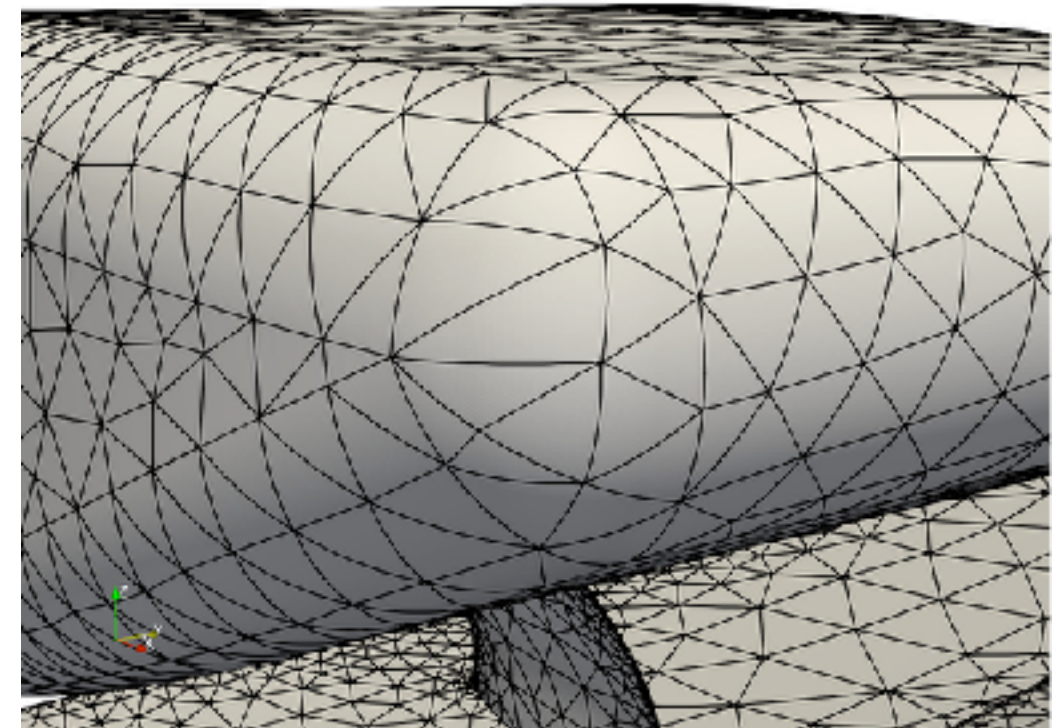
$\mathbf{P}_k(\mathbf{x}_j)$ is the projection of the vertex on surface ID k



Mesh curving on a surface



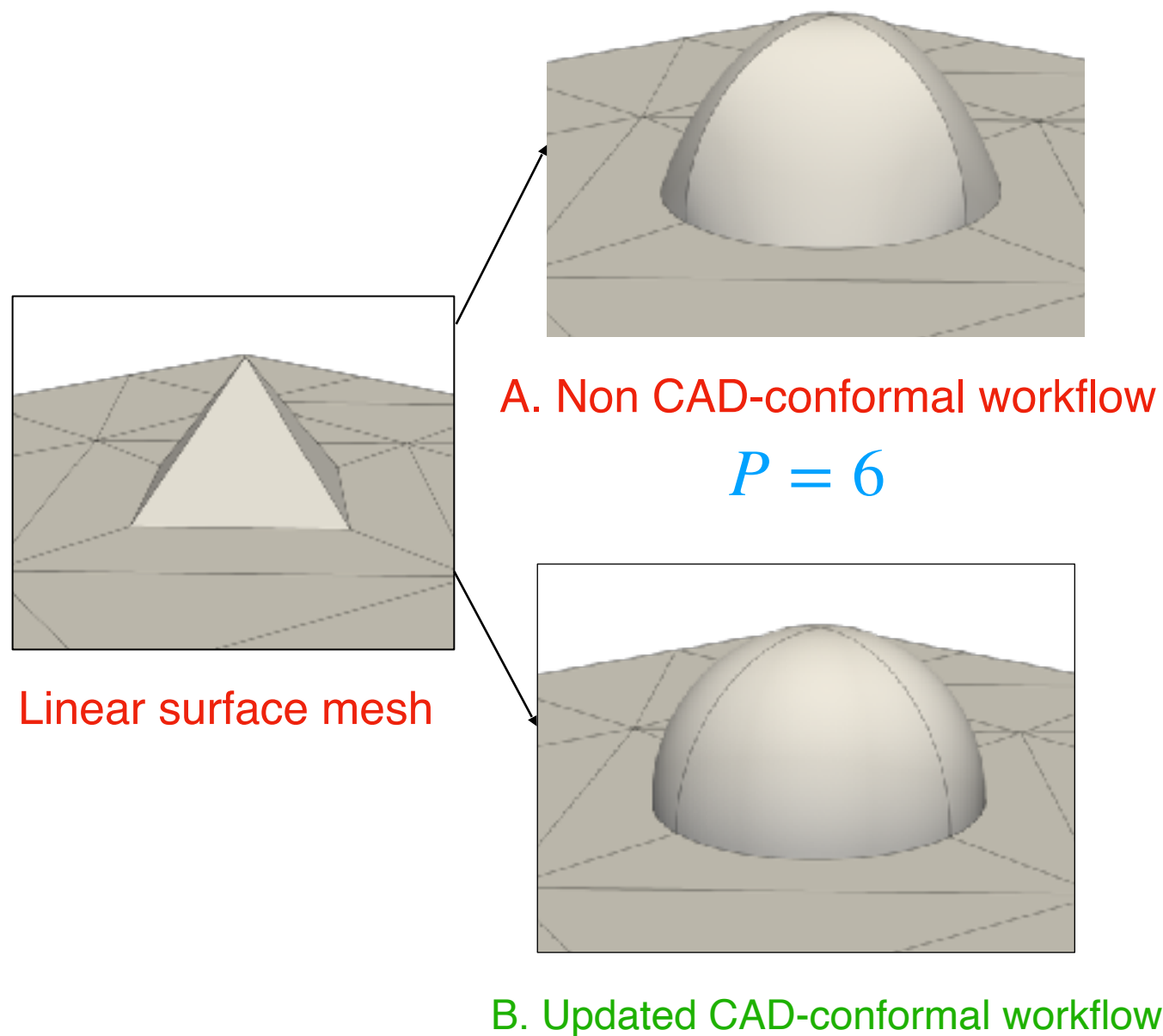
Closest CAD projection



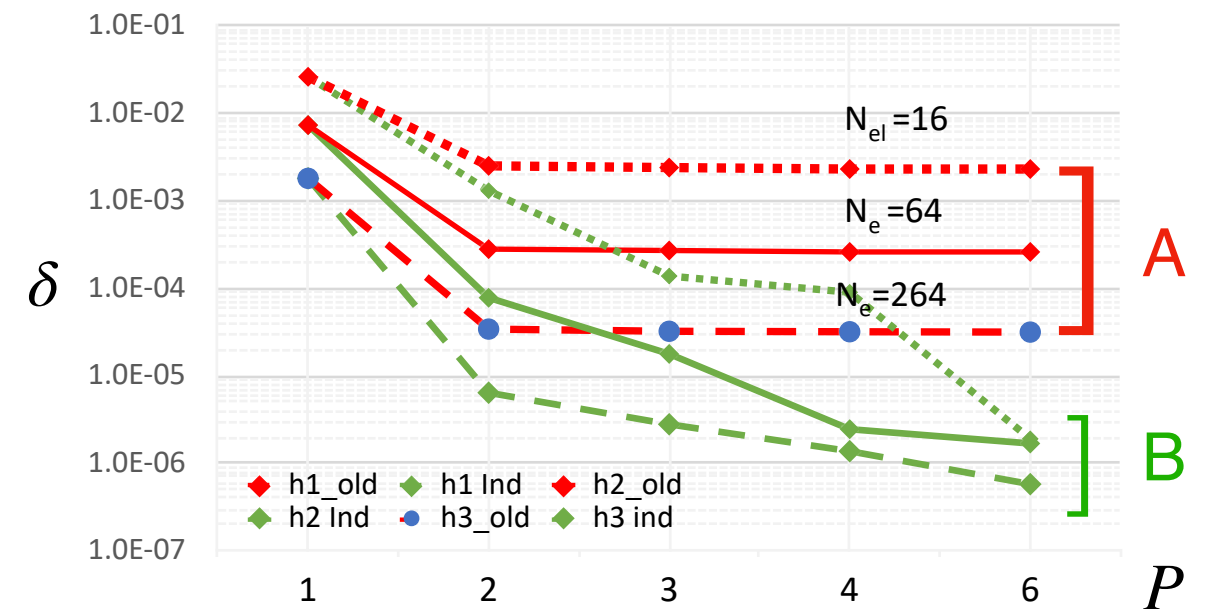
Optimised

Performance assessment

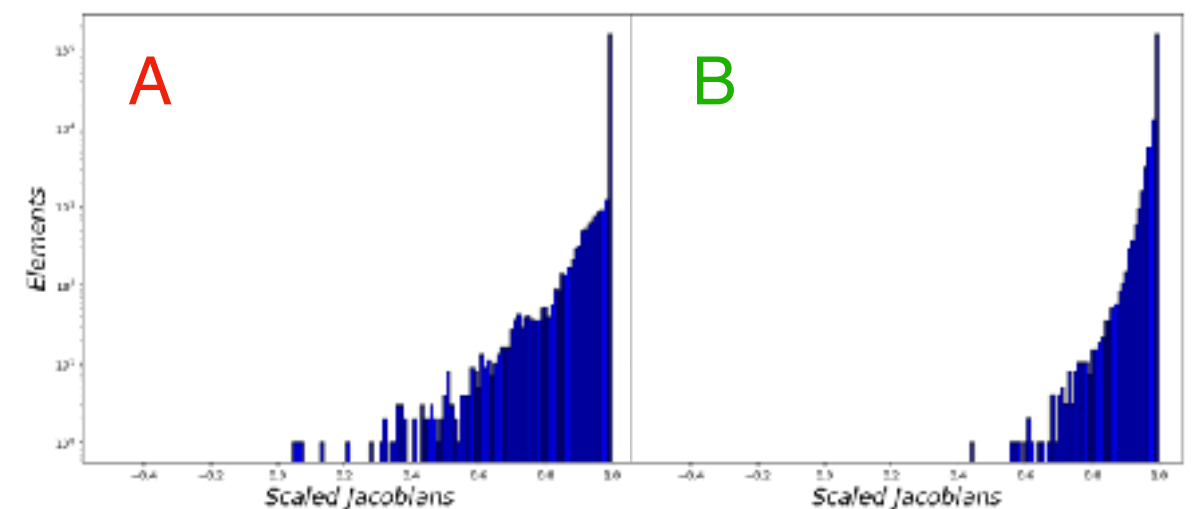
Hemisphere domain test case



Geometrical accuracy



Mesh quality

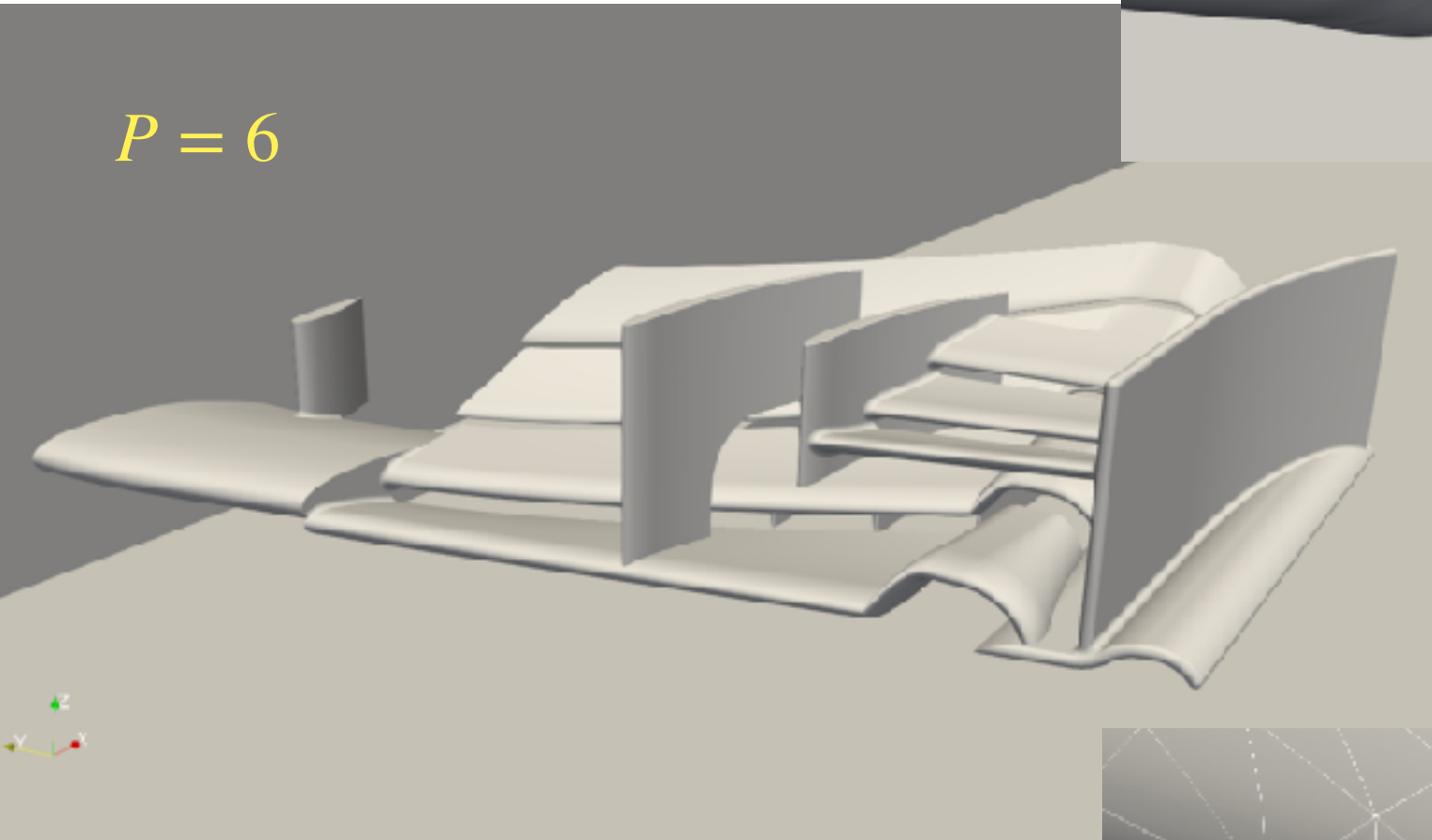


Perring F1 front wing

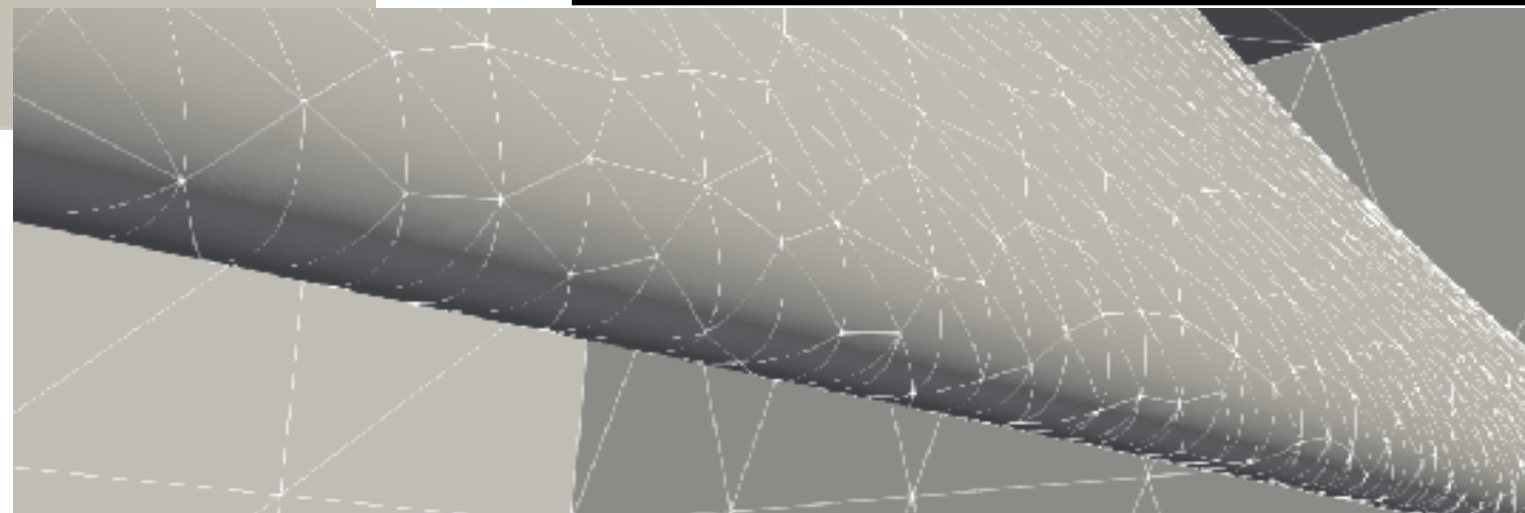
Non-CAD conformal high-order mesh

50k Surface Elements

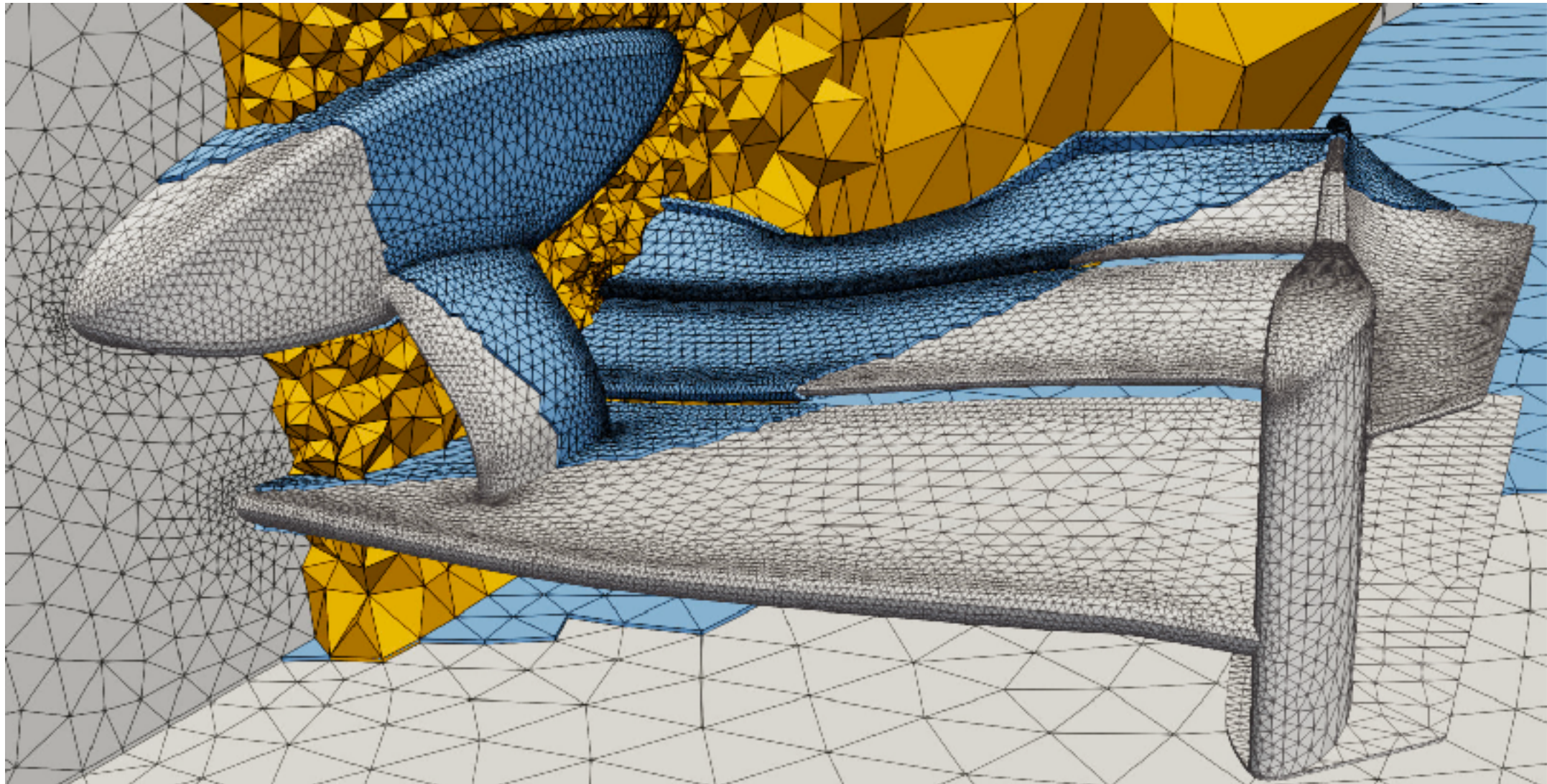
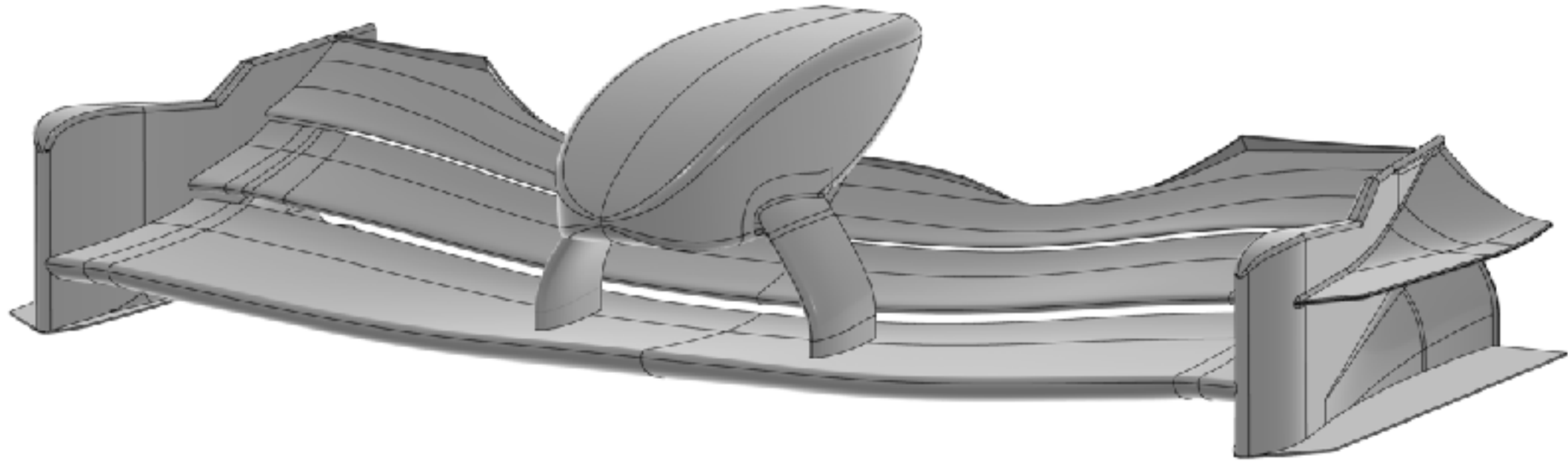
$P = 6$



CAD-conformal high-order mesh

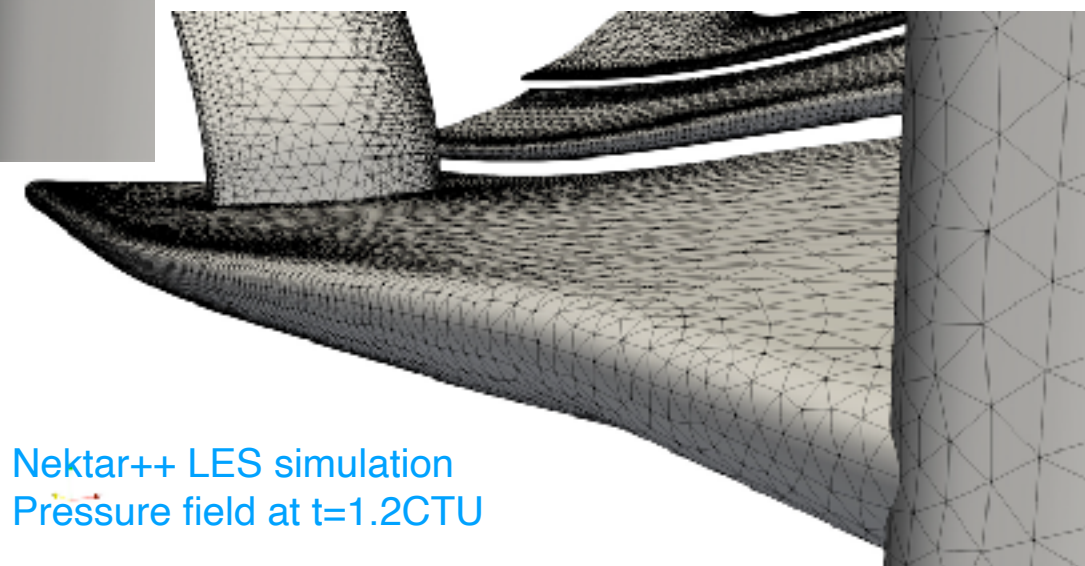
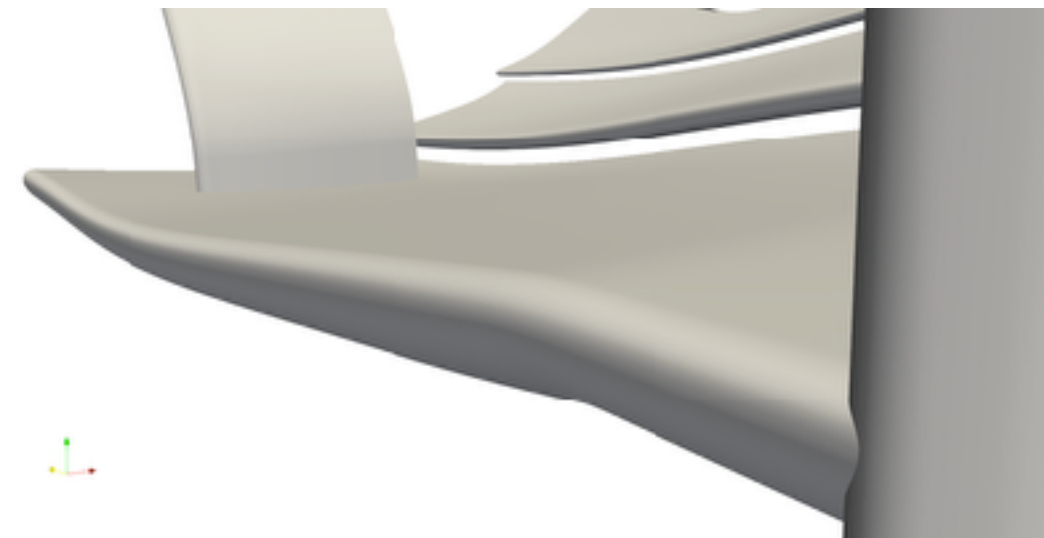
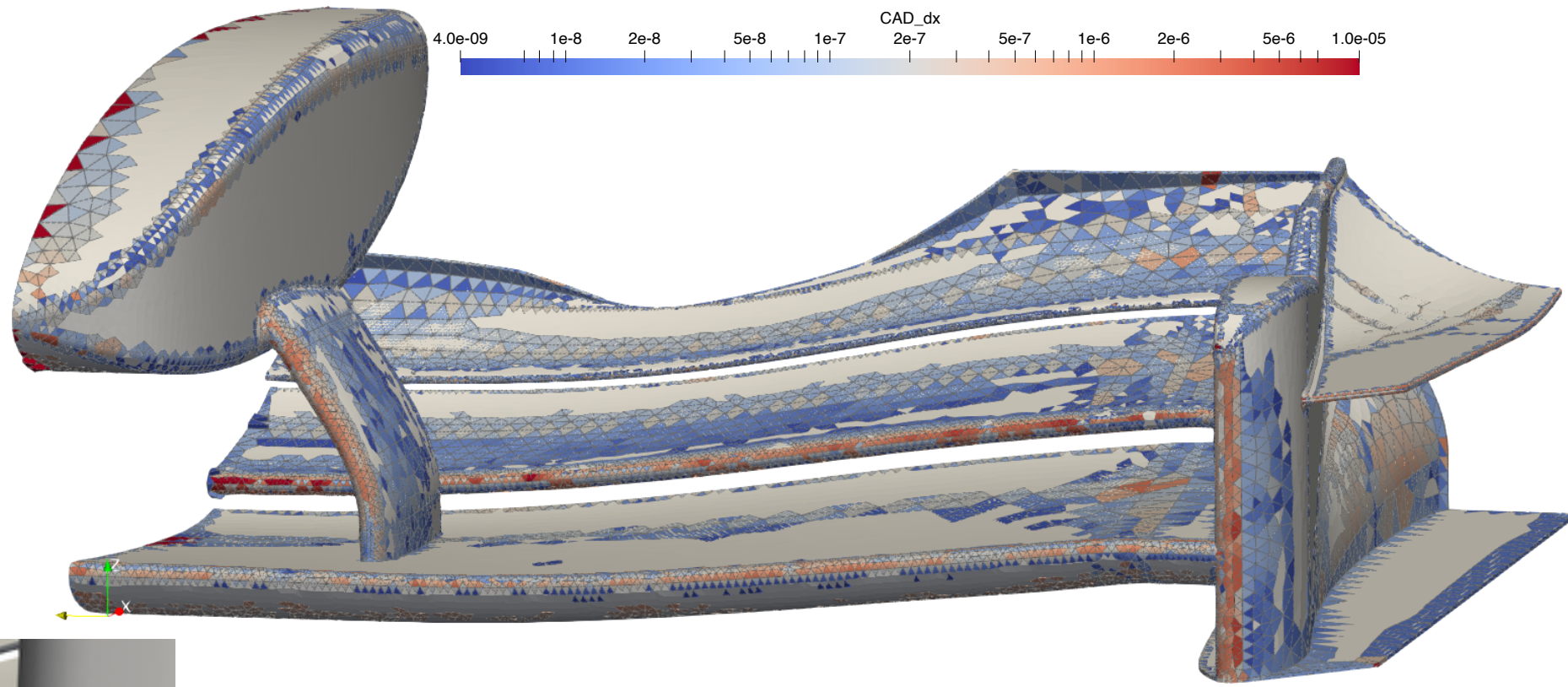
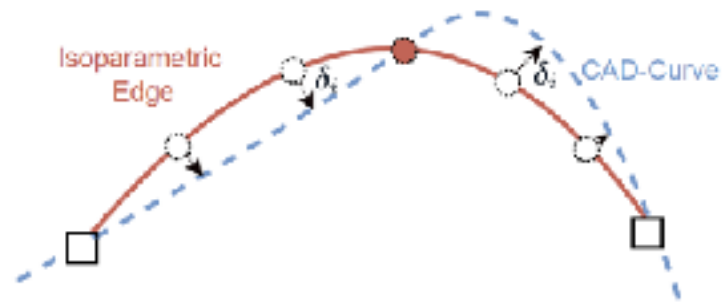


Imperial front wing geometry



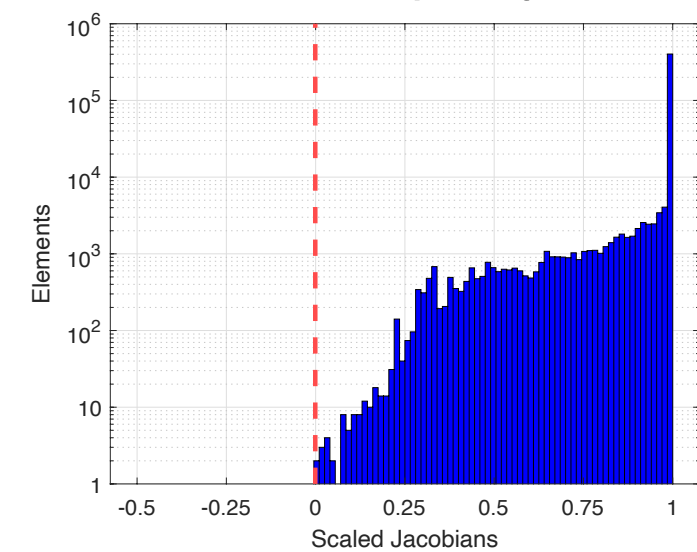
IFW assessment

Geometrical accuracy



Nektar++ LES simulation
Pressure field at $t=1.2CTU$

Mesh quality



Final comments

- Benefits from the robustness of third-party linear mesh generators.
- Incorporates a multi-patch option not currently available in Nekmesh.
- Facilitates the assessment of geometrical accuracy of both linear and high-order meshes.
- Robustness of high-order meshing significantly improved but still work to be done.