Tetrahedron VII

BSC-UPC
Barcelona

Oct 9-11 2023

Monge-Ampère Gravity

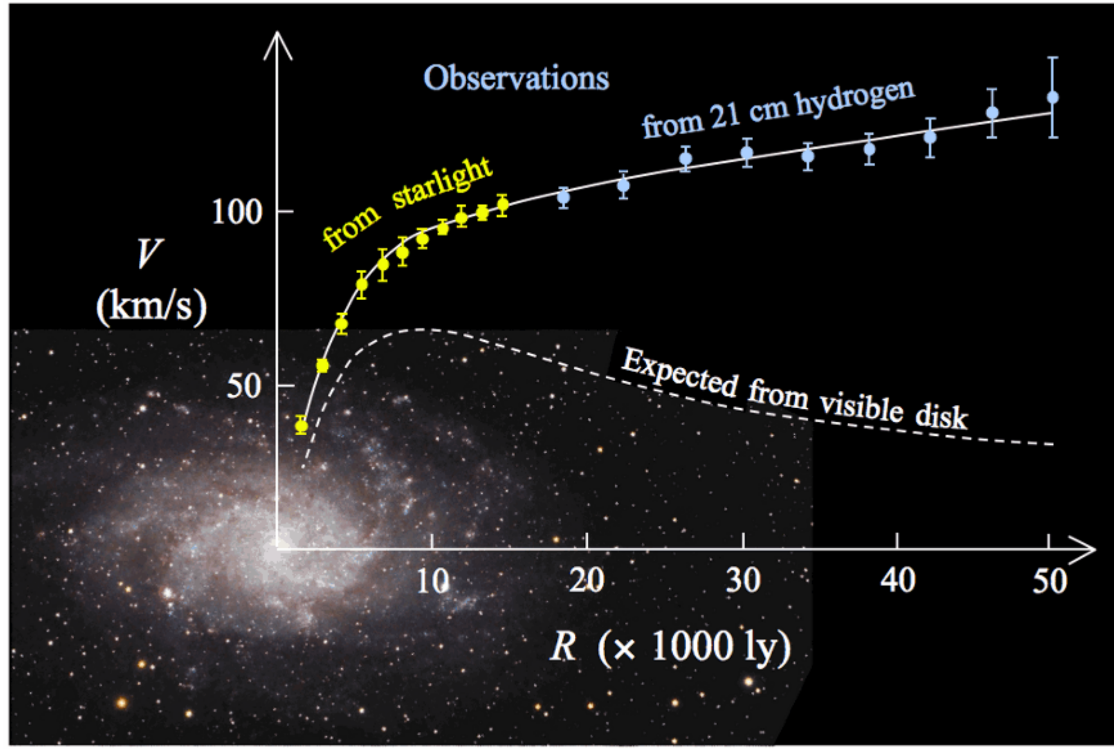
Bruno Lévy
ParMA project-team

Joint work with Yann Brenier, Pierre Boldrini and Roya Mohayaee

Inria Saclay
Labo. de Mathématiques d'Orsay
Université Paris Saclay

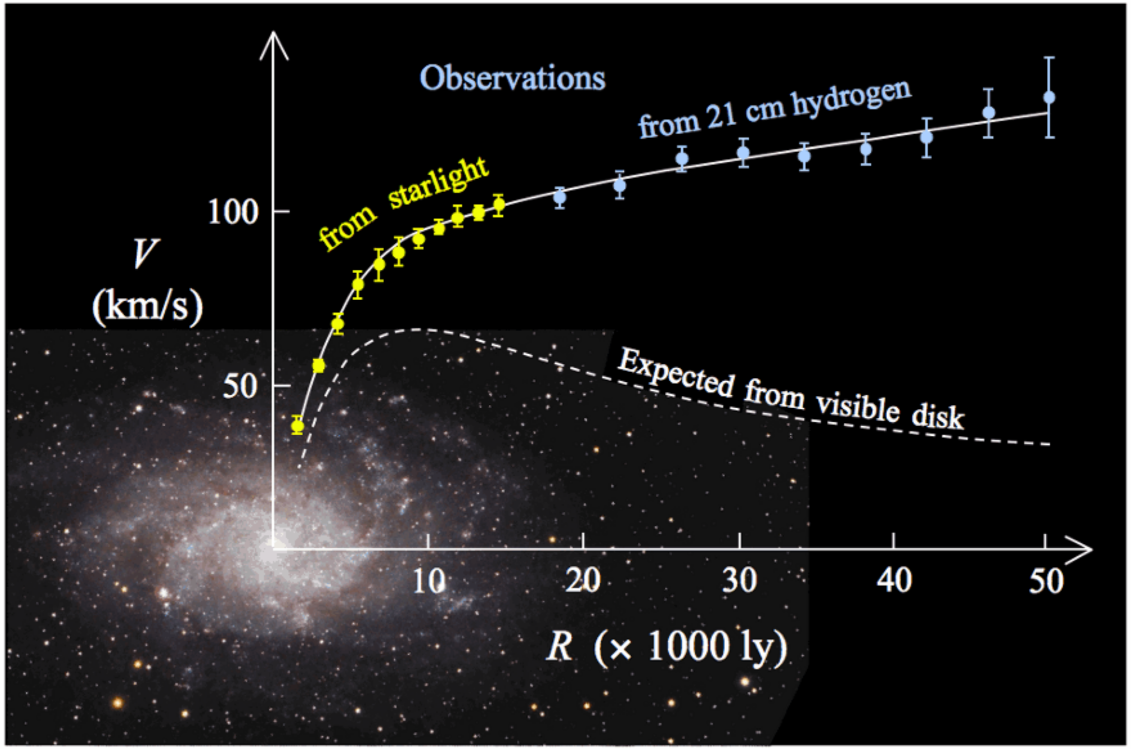
Mysteries in the sky …

Mysteries in the sky



Vera Rubin - 1962

Mysteries in the sky



Vera Rubin - 1962

There is more mass than what we observe

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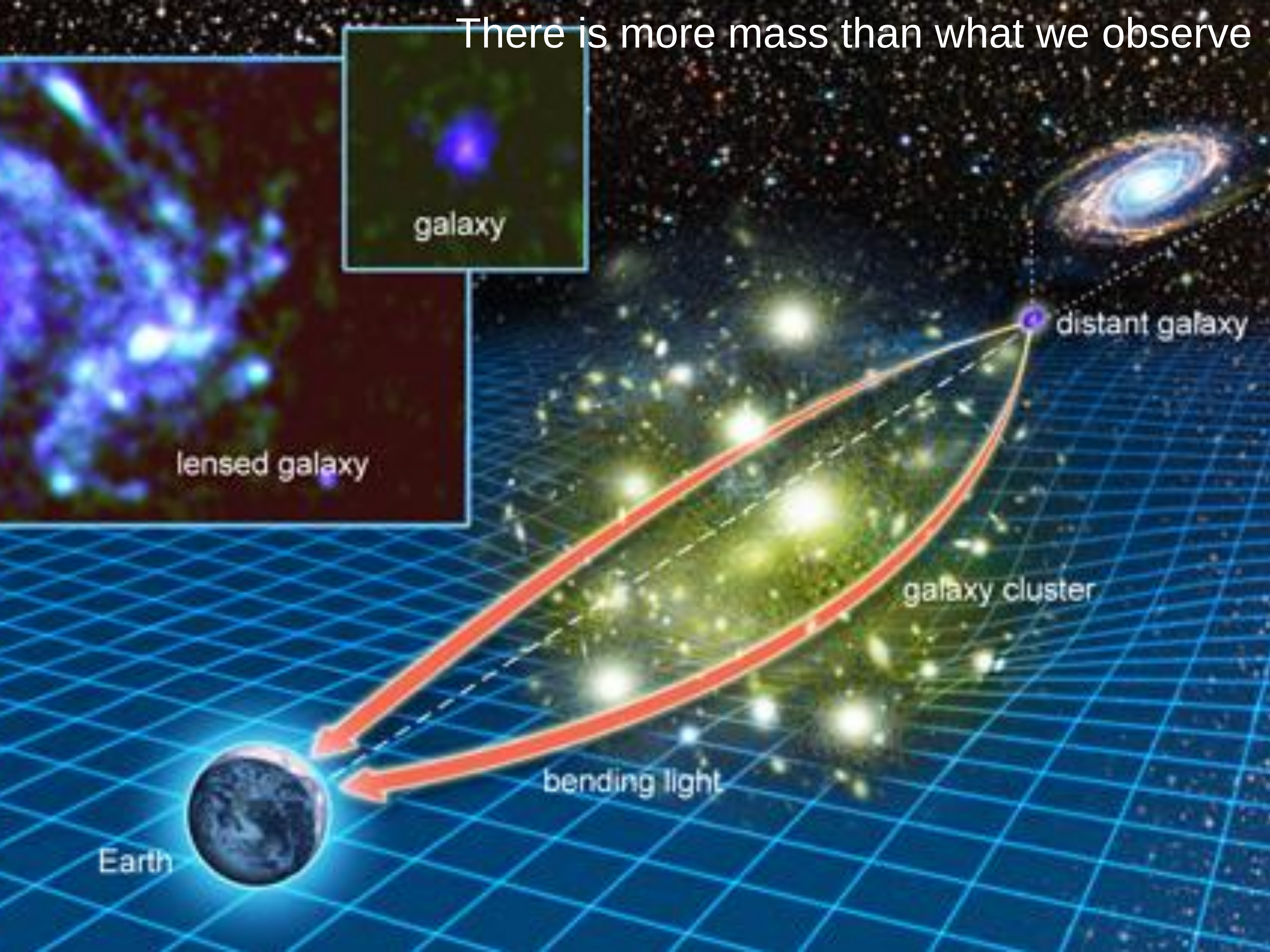
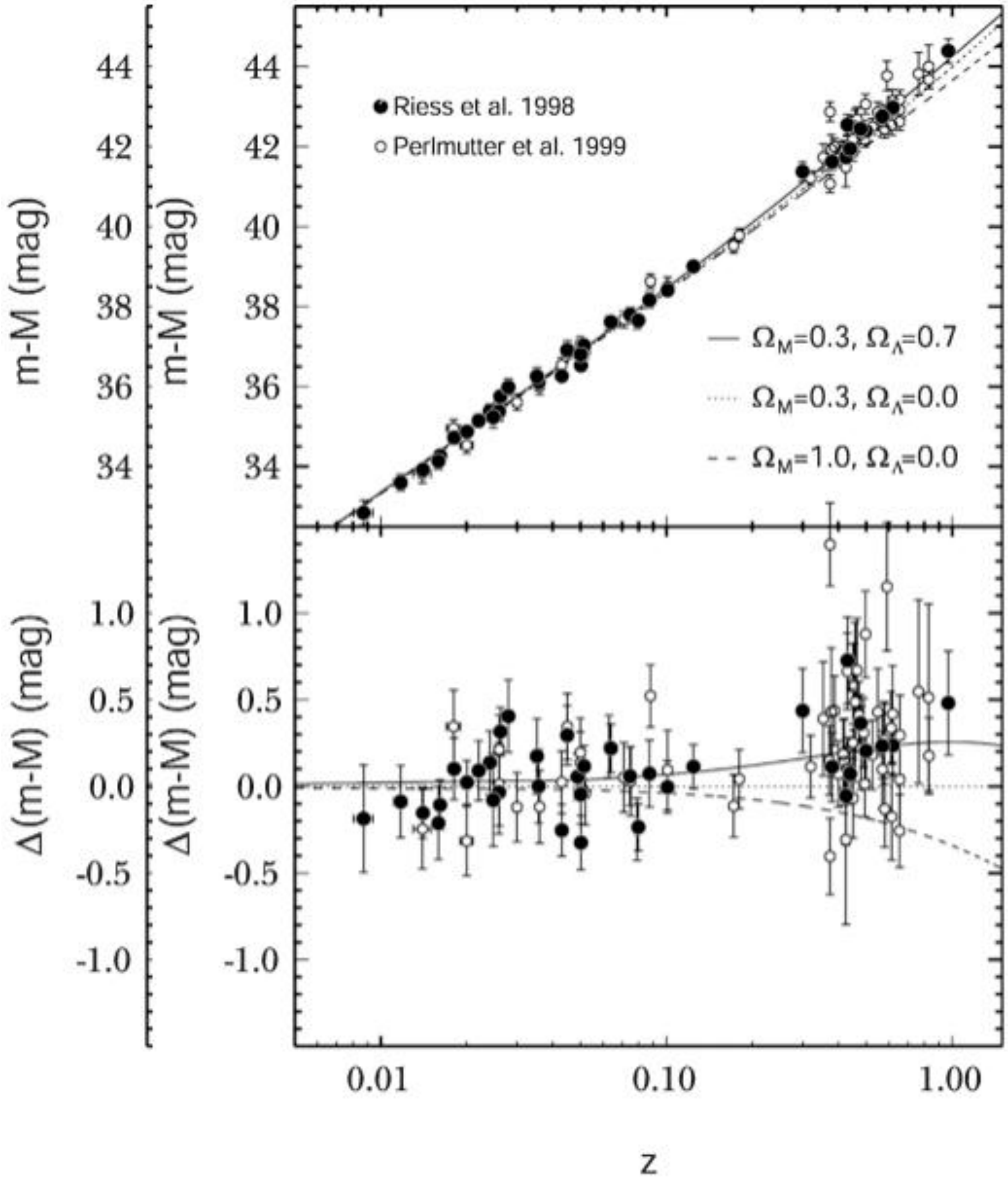
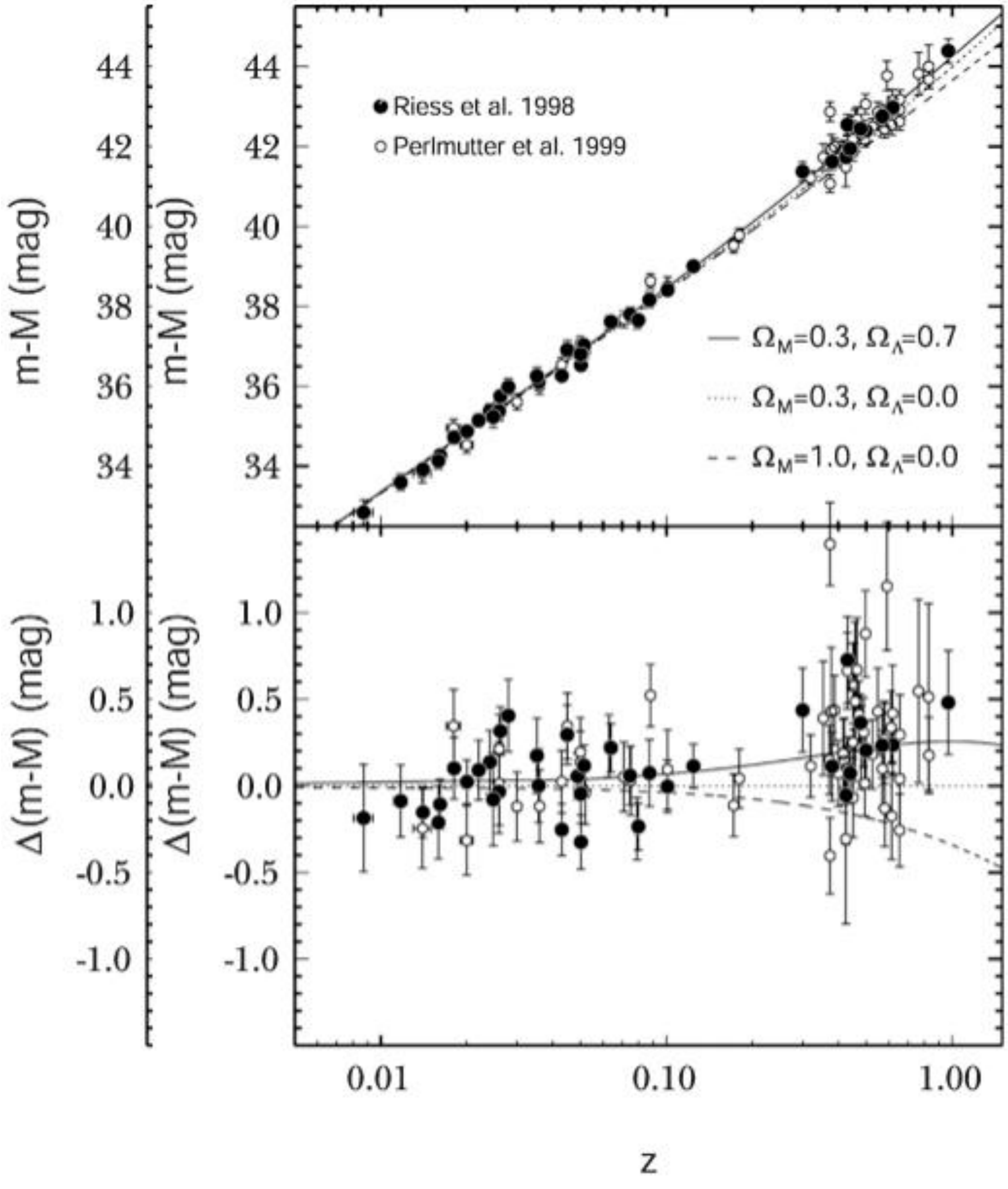




Type Ia supernovae "standard candles"

Permutter
Riess



Type Ia supernovae "standard candles"

Permutter
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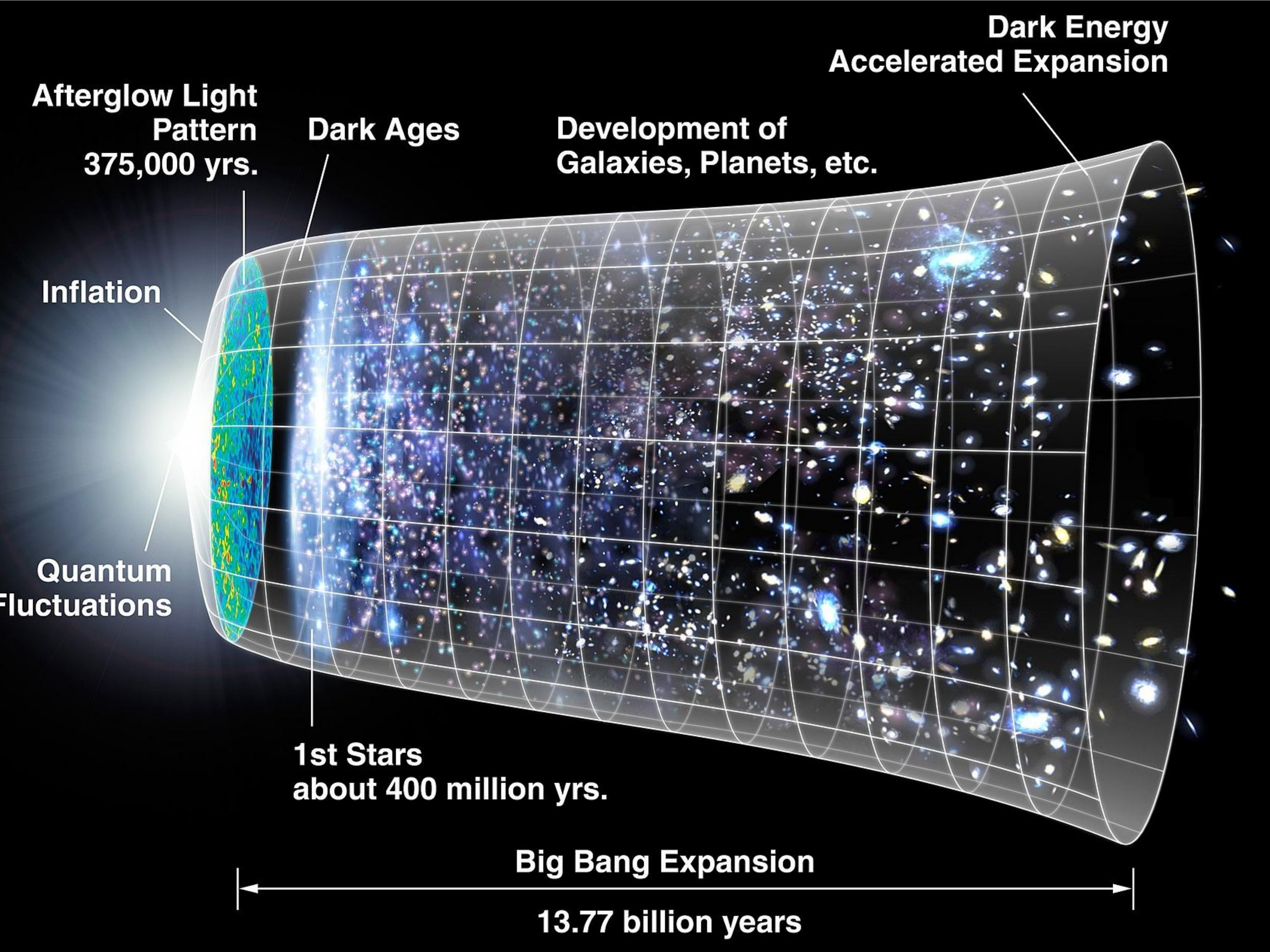
The expansion of the Universe is accelerating.


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years

Mysteries in the sky

- There seems to be more matter than what we observe…

- The big-bang is big-banging faster than we thought …

Mysteries in the sky

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“dark matter” (but we do not know what it is)

- The big-bang is big-banging faster than we thought …

“dark energy” (but we do not know what it is)

Mysteries in the sky

Anomalies and tensions in Λ-CDM (Review in [Peebles 2022])

- Baryonic Tully-Fischer rotation curve
- Acceleration of the expansion
- Anomalous abundance of small haloes
- Formation time of structures
- Anomalous dipole
- Anomalous bulk flow
- …

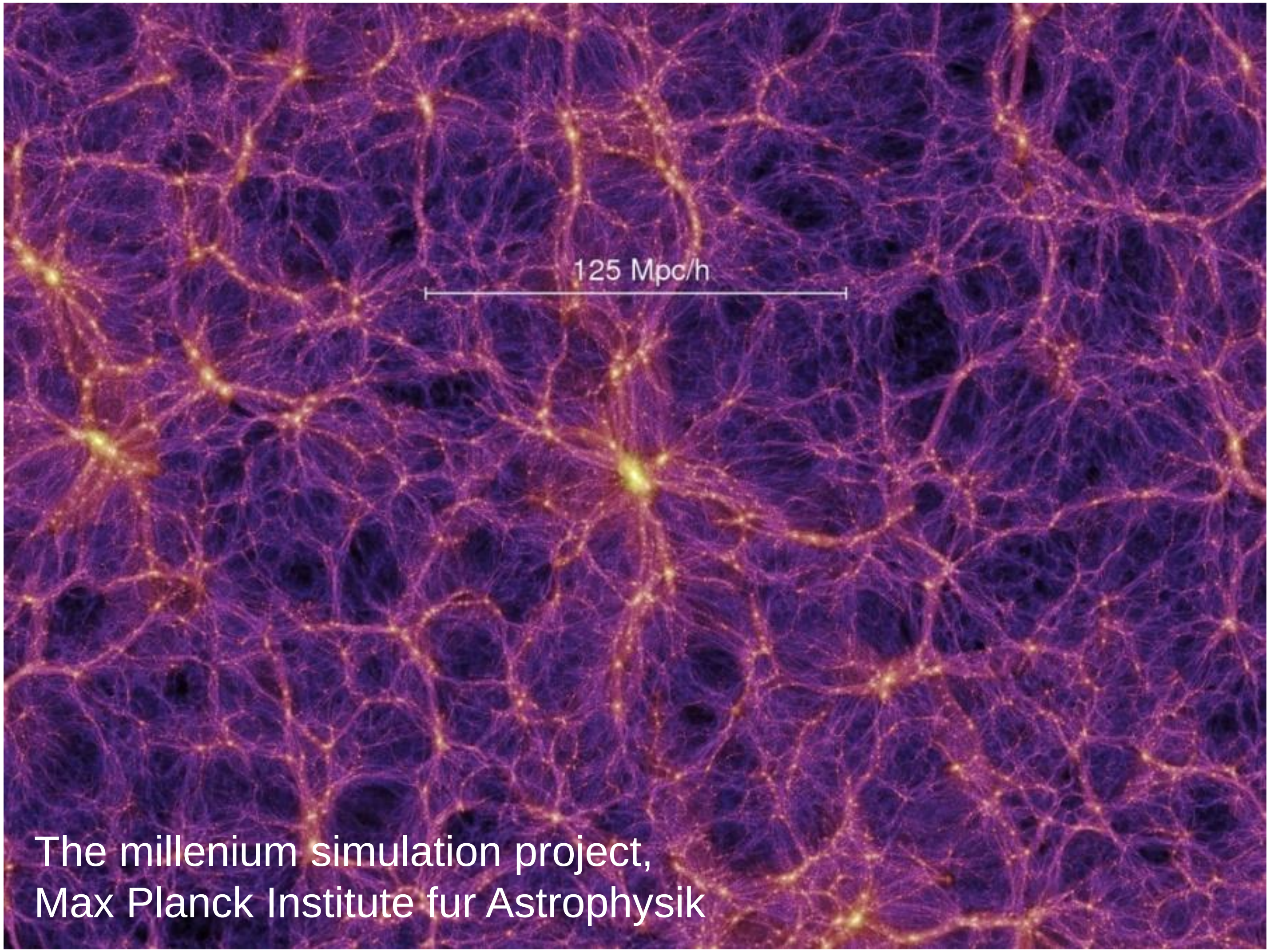
Mysteries in the sky

Anomalies and tensions in Λ CDM (Review in [Peebles 2022])

- Baryon
- Accele
- Anoma
- Format
- Anoma
- Anoma
- …

We need new ideas,
new models,
new equations here !

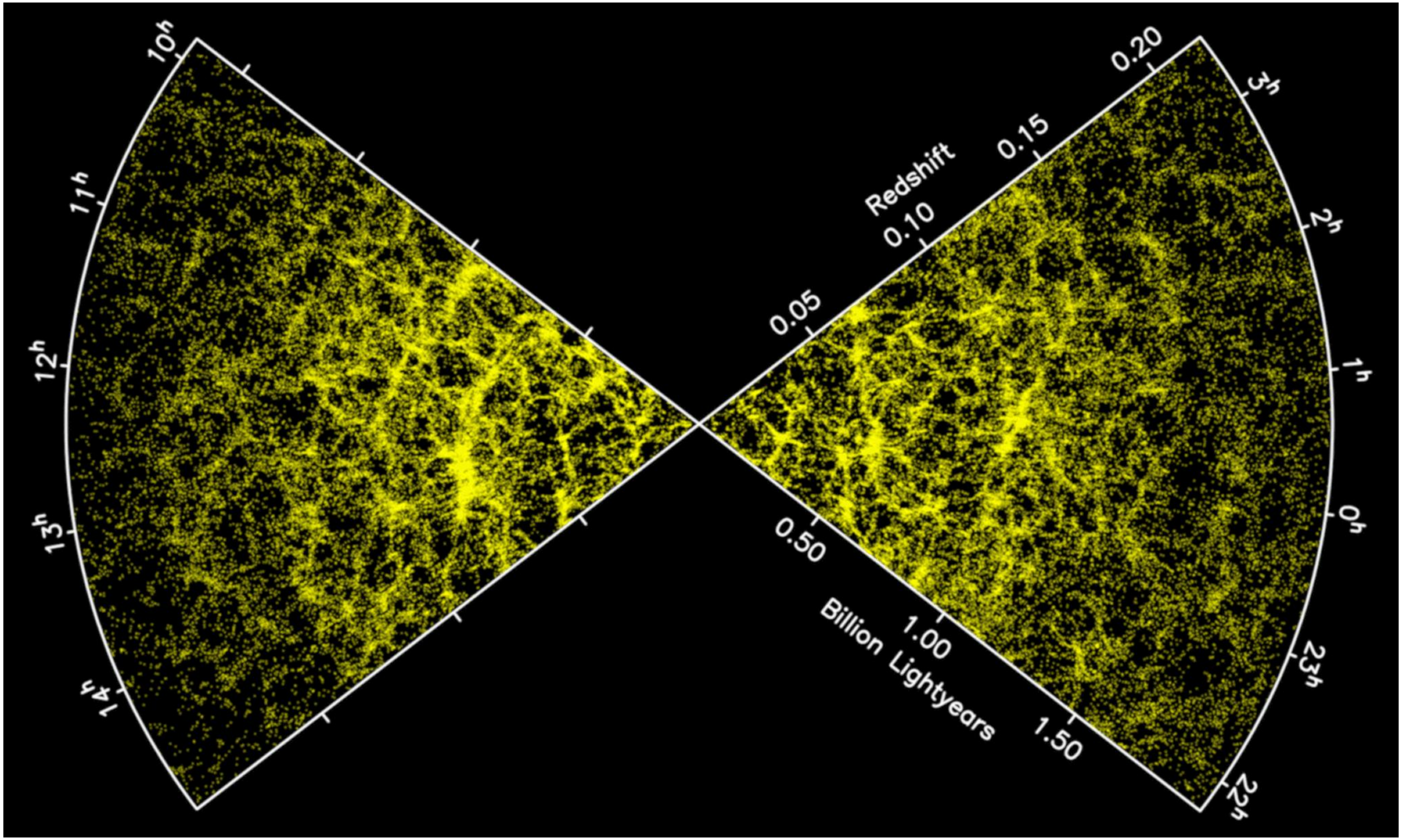
Simulations



The millenium simulation project,
Max Planck Institute fur Astrophysik

pc/h : parsec (= 3.2 light year)

Observations



1. Newton

$$F = -\mathcal{G}\frac{m_1 m_2}{|\mathbf{r}_2 - \mathbf{r}_1|^2}$$

$$\left\{ \begin{array}{lcl} F & = & \nabla \phi \\ \Delta \phi & = & 4\pi\mathcal{G}(\rho - \bar{\rho}) \end{array} \right.$$

2. Brenier-Monge-Ampère

$$\left\{ \begin{array}{lcl} F & = & \nabla \Phi \\ \mathbb{\Delta}\Phi & = & \frac{\rho}{\bar{\rho}} \\ \Phi & = & \frac{\phi}{4\pi\mathcal{G}\bar{\rho}} + \frac{|\mathbf{r}|^2}{2} \end{array} \right.$$

3. Optimal Transport

$$T \quad = \quad \nabla \Phi$$

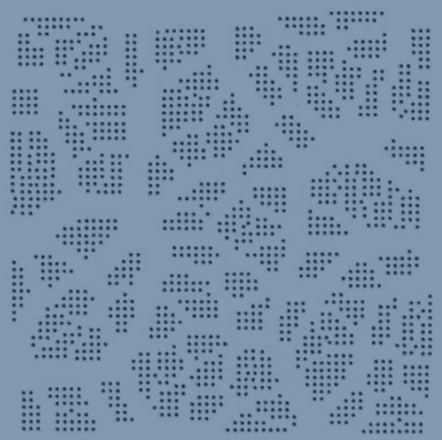
$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r} \right]$$

subject to:

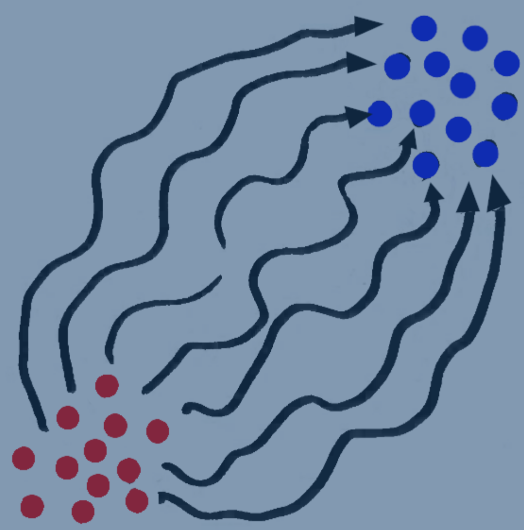
$$\int_B \bar{\rho}\, d\mathbf{q} = \int_{T^{-1}(B)} \rho(\mathbf{r}) d\mathbf{r} \quad \forall B$$

4. Discrete Optimal Transp.

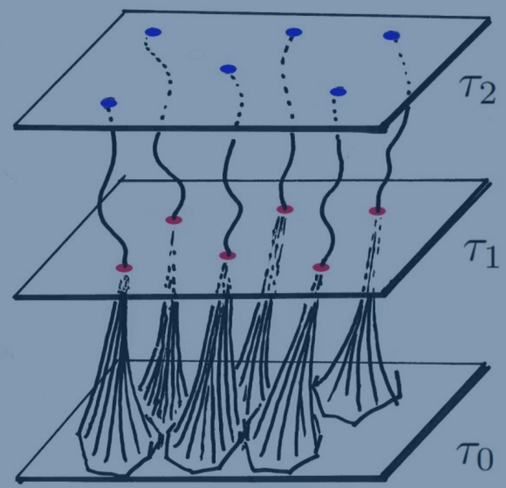
$$\inf_{\sigma \in S_N} \left[\left| \mathbf{r}_i - \mathbf{q}_{\sigma(i)} \right|^2 \right]$$



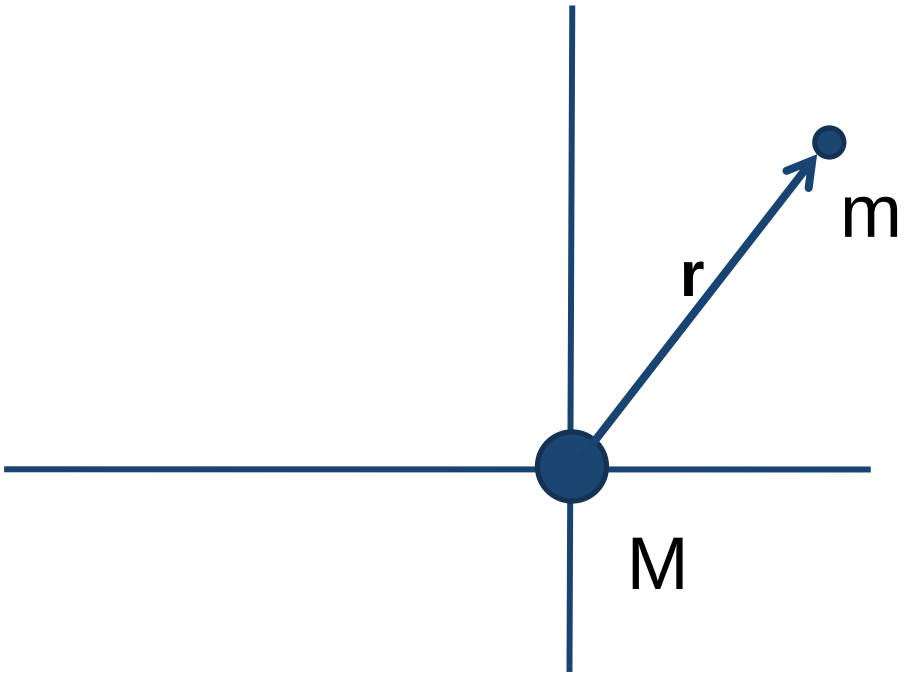
5. Large Deviations Pple.



6. The Path Bundle Method



1. Newton-Poisson



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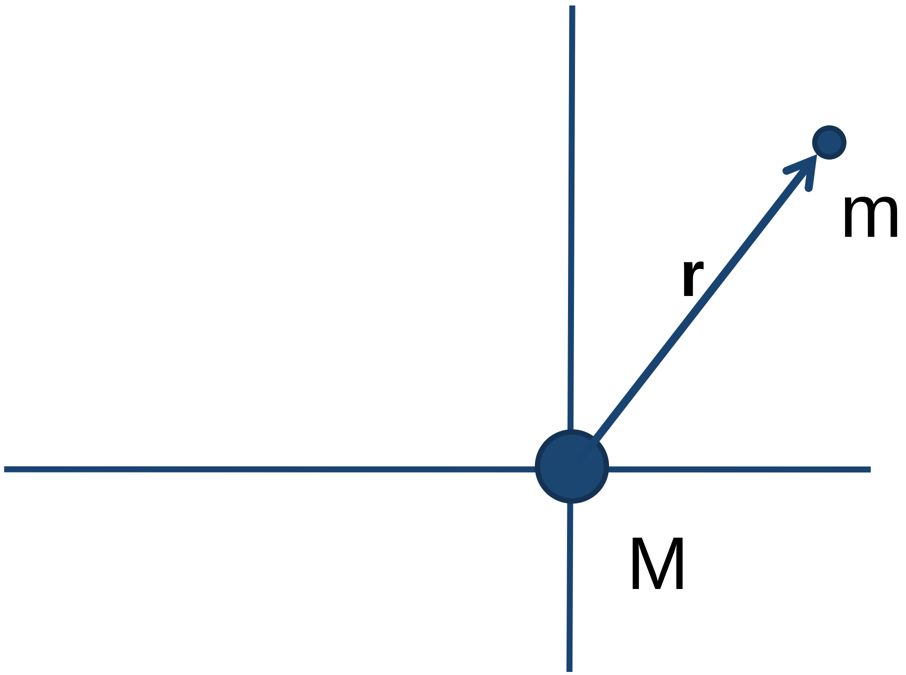

$$\mathbf{F}(\mathbf{r}) = m\mathbf{G}(\mathbf{r}) = -m\mathcal{G}M\frac{\mathbf{r}}{\|\mathbf{r}\|^3}$$

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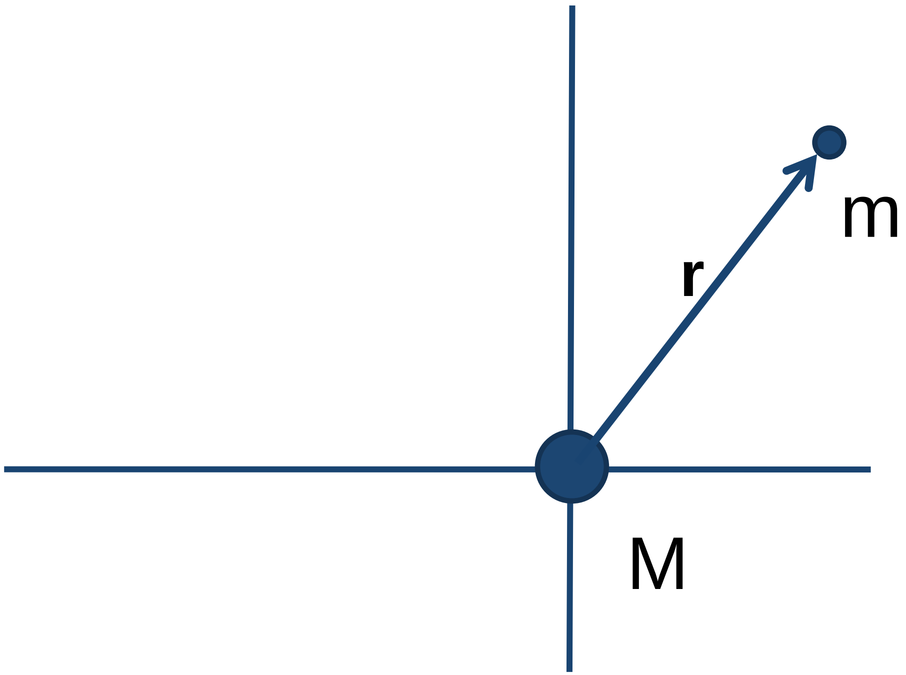

$$\mathbf{F}(\mathbf{r}) = m\mathbf{G}(\mathbf{r}) = -m\mathcal{G}M\frac{\mathbf{r}}{\|\mathbf{r}\|^3}$$

$$\mathbf{G}(\mathbf{r}) = -\nabla\phi(\mathbf{r}) \quad ; \quad \phi(\mathbf{r}) = -m\mathcal{G}\frac{M}{\|\mathbf{r}\|}$$

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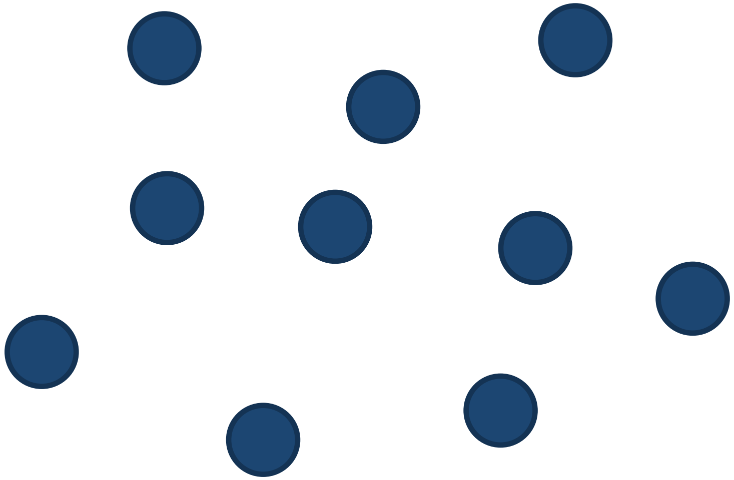
$$\mathbf{F}_i = m_i \mathbf{G}_i$$



$$\mathbf{G}_i = -\mathcal{G} \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\mathbf{x}_i - \mathbf{x}_j}{\|\mathbf{x}_i - \mathbf{x}_j\|^3}$$

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$$\mathbf{G}_i = \nabla \phi_i \quad ; \quad \phi_i = -\mathcal{G} \sum_{\substack{j=1 \\ j \neq i}} \frac{m_j}{\|\mathbf{x}_i - \mathbf{x}_j\|}$$

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(F = ma)

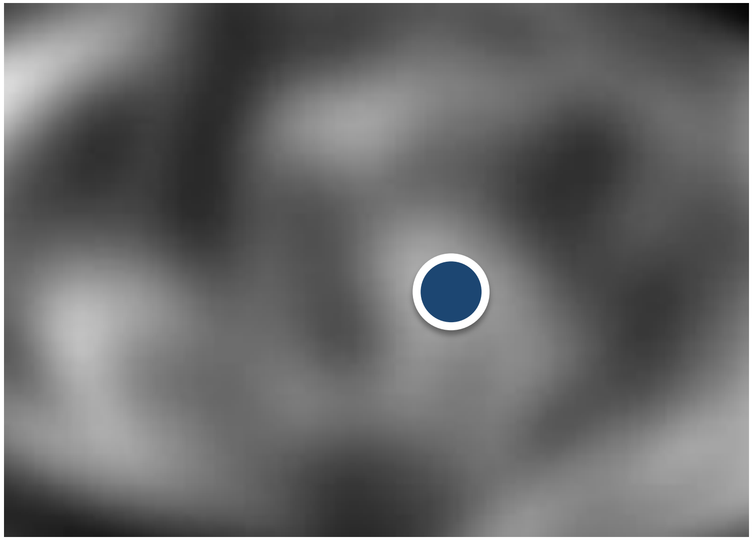
Gravity for a set of particles
(N-body)
Lagrangian coordinates

1. Newton-Poisson

$\rho(\mathbf{x}, t)$

Gravity for a density field ?
Eulerian coordinates

1. Newton-Poisson



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Gravity for a density field ?
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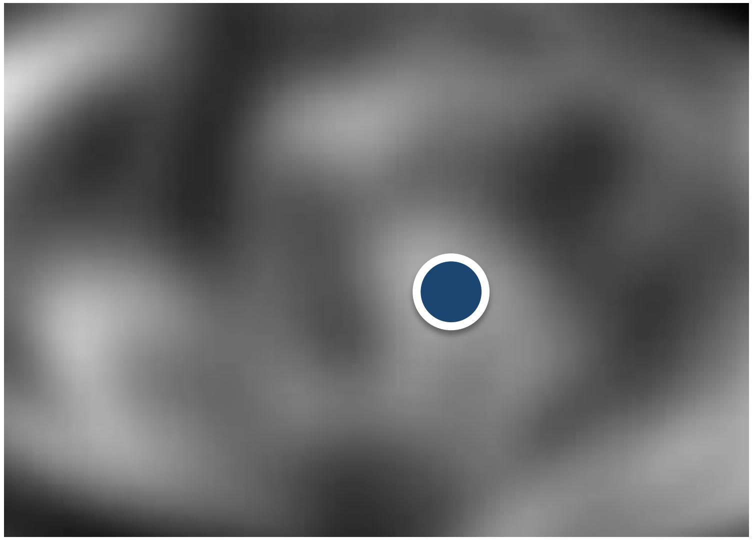

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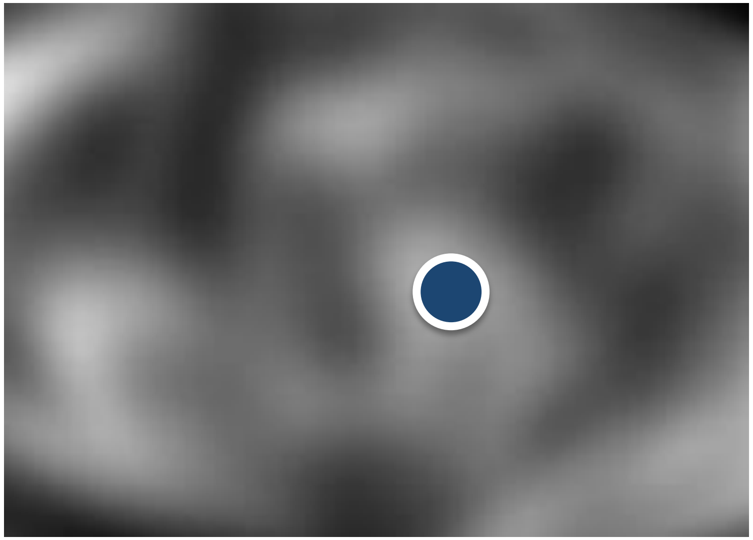

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Green function

$$\Delta f = g$$

$$f(\mathbf{x}) = \iiint_V K(\mathbf{x}, \mathbf{y}) g(\mathbf{y}) d\mathbf{y}$$

$$K(\mathbf{x}, \mathbf{y}) = -\frac{1}{4\pi} \frac{1}{\|\mathbf{x} - \mathbf{y}\|}$$

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$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla\phi$$

Velocity field

Correction term
(convective derivative)

$$\Delta\phi = 4\pi\mathcal{G}\rho$$

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(Mass conservation *continuity eqn*)

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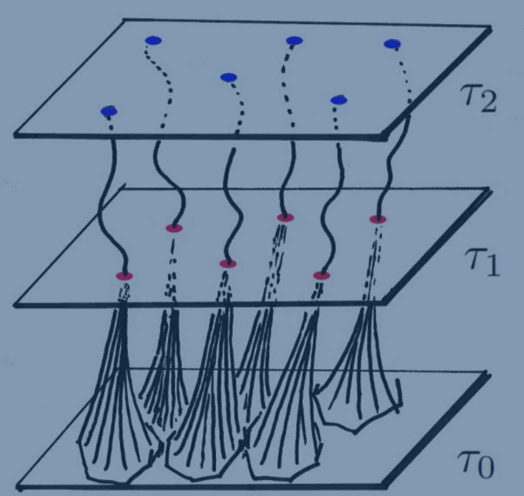

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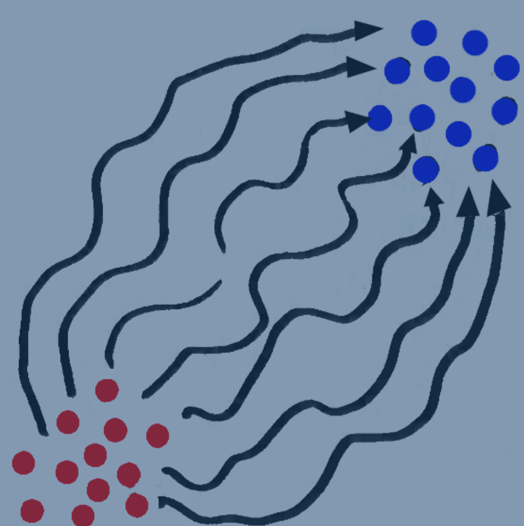

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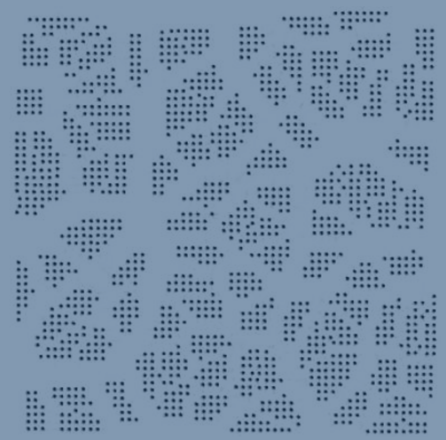

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Taylor expansion of the determinant of a matrix around the identity:

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Newton-Poisson

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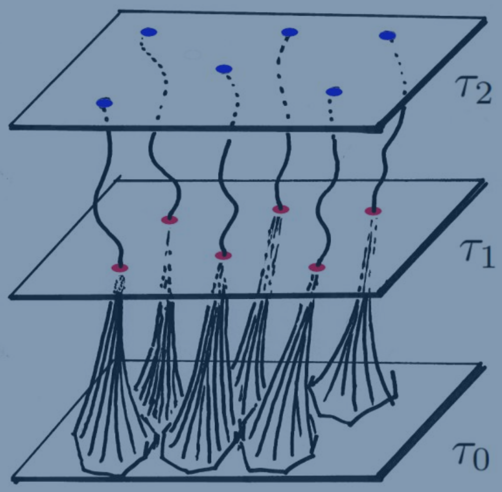

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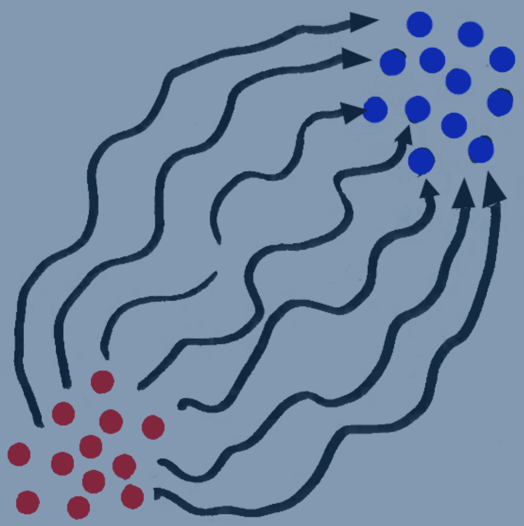

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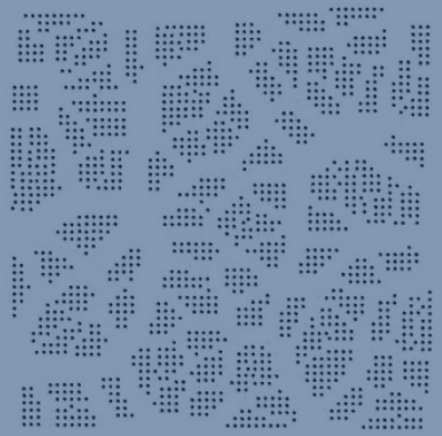

3. Optimal Transport and Monge-Ampère

$$\Delta \Phi = \frac{\rho}{\bar{\rho}}$$

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$$\bar{\rho}\Delta\Phi = \rho$$

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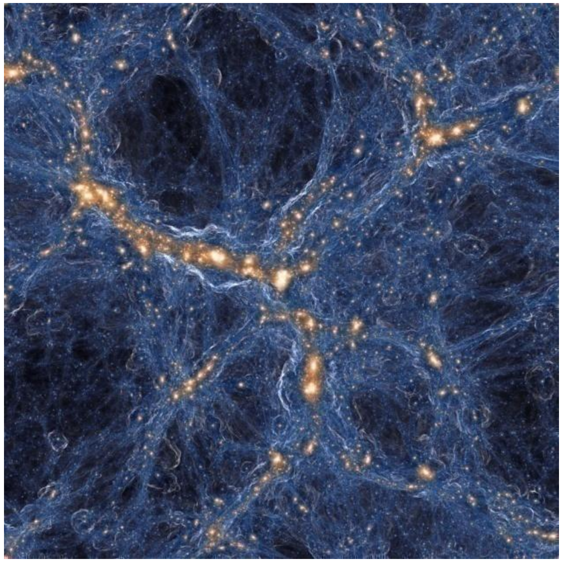

ρ

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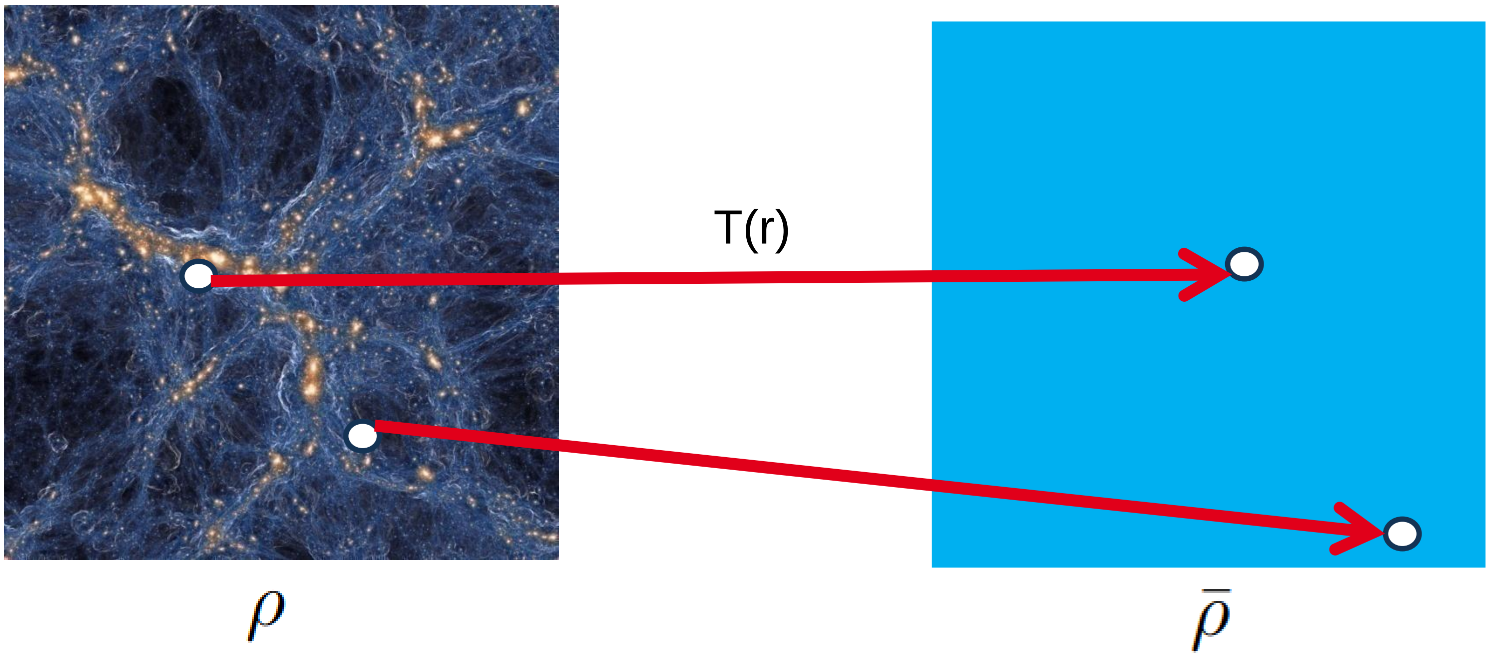

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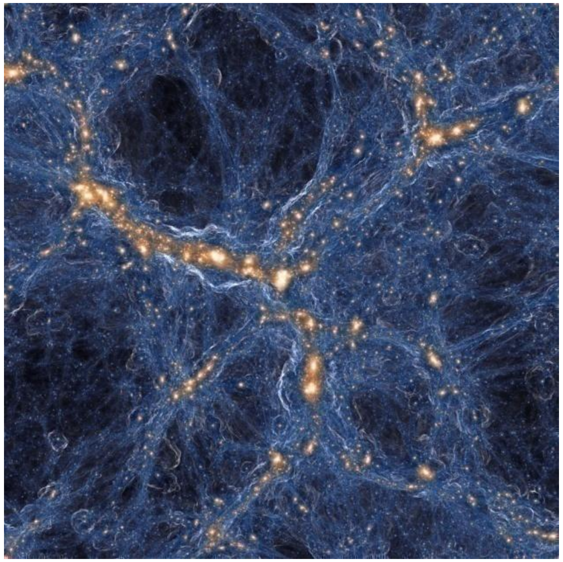

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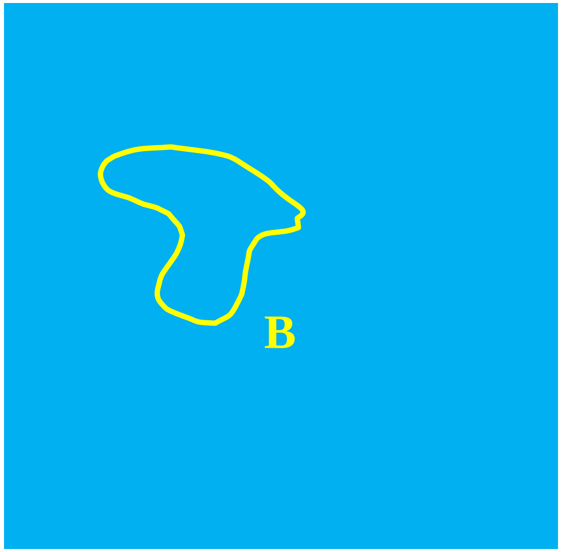

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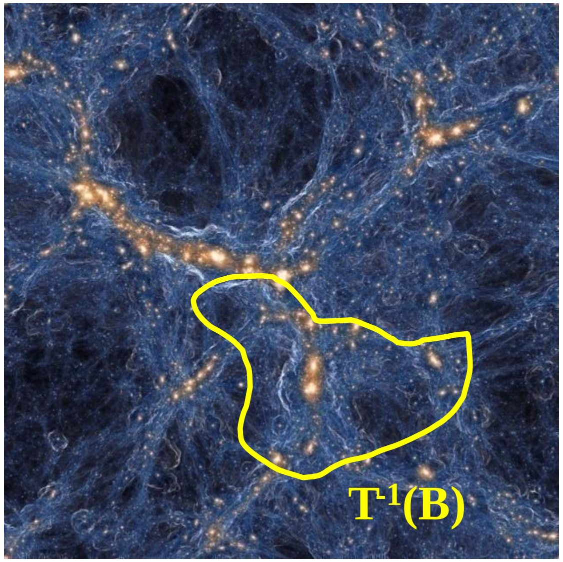

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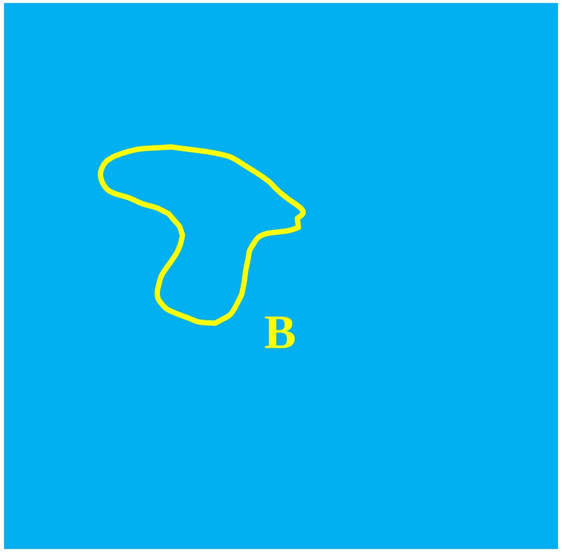

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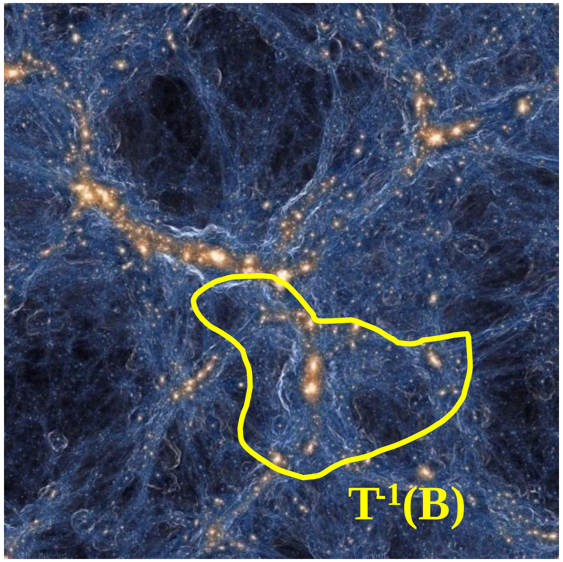

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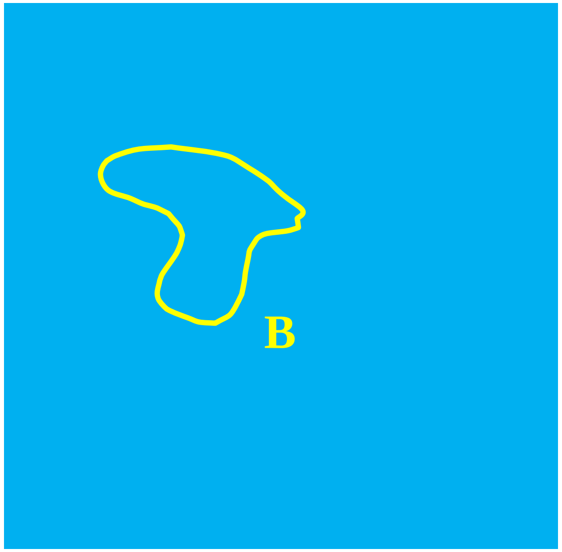

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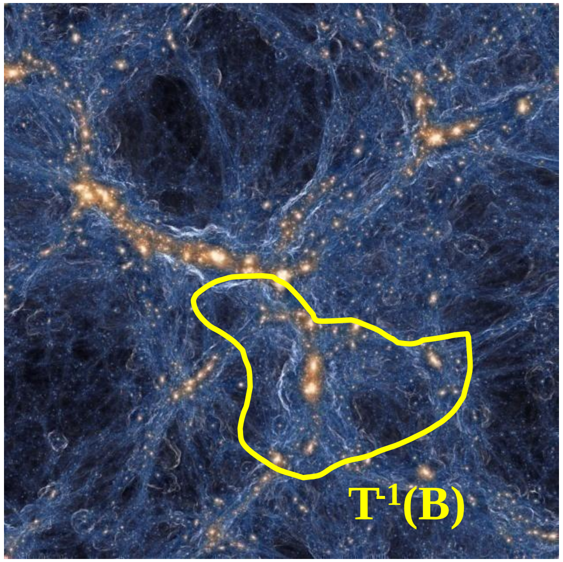
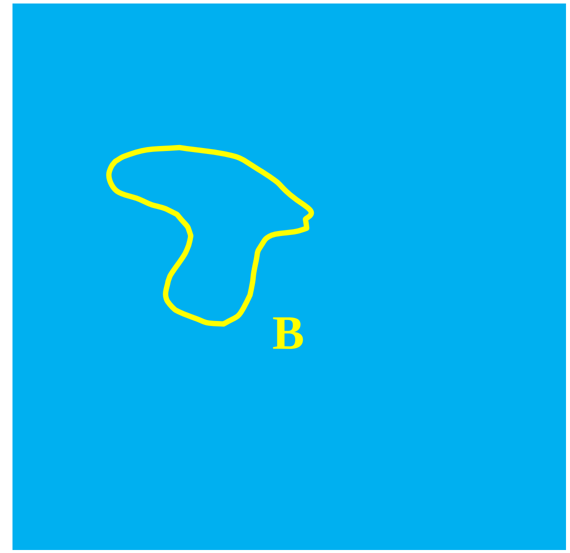
subject to:

$$\int_B \bar{\rho} d\mathbf{q} = \int_{T^{-1}(B)} \rho(\mathbf{r}) d\mathbf{r} \quad \forall B$$

3. Optimal Transport and Monge-Ampère



ρ



$\bar{\rho}$

$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r} \right]$$

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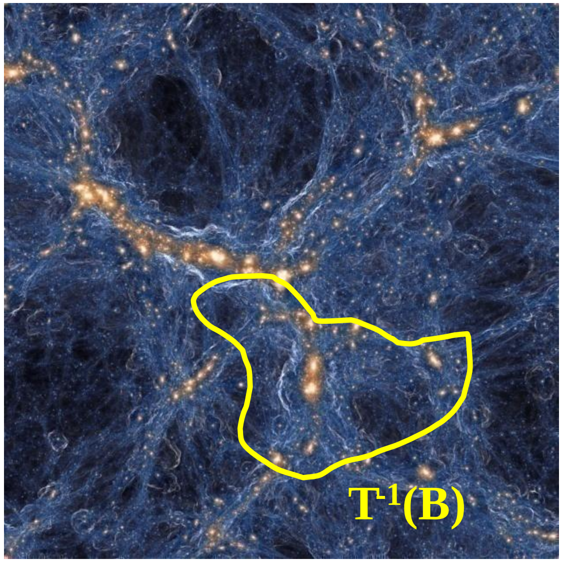
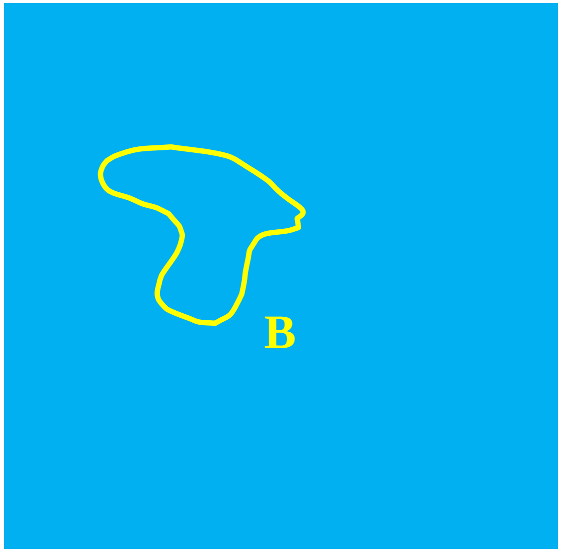


ρ



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$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r} \right]$$

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$$\sup_T \inf_\Psi \Big[\ \mathcal{L}(T, \Psi) = \int \rho(\mathbf{r}) T(\mathbf{r}) \cdot \mathbf{r} d\mathbf{r} + \int \bar{\rho} \Psi(\mathbf{q}) d\mathbf{q} \ - \int \Psi(T(\mathbf{r})) \rho(\mathbf{r}) d\mathbf{r} \ \Big]$$

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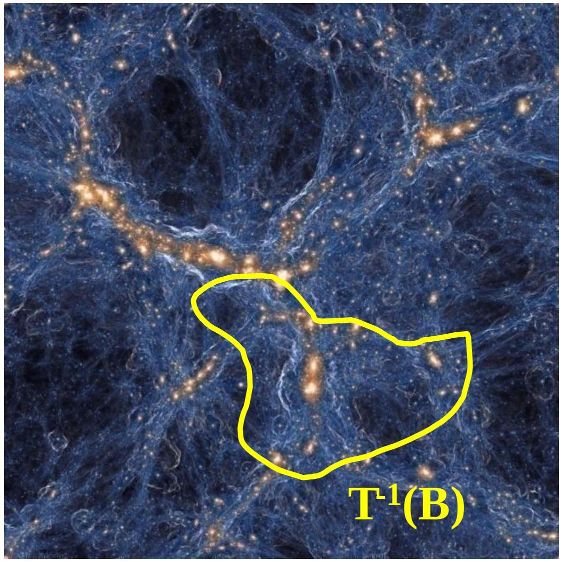

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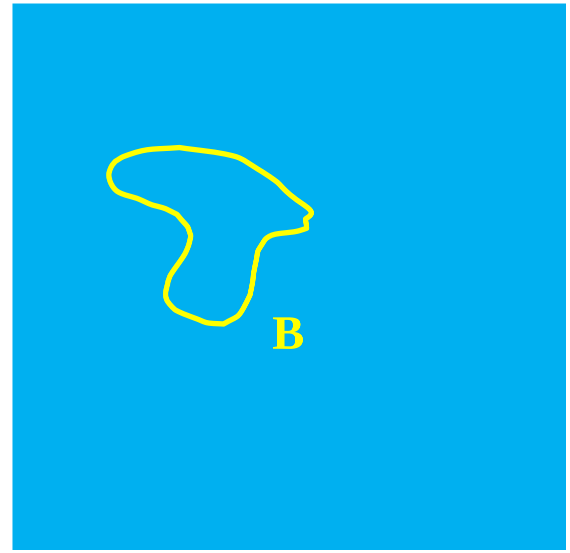

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Lagrange multiplier associated with the constraint

3. Optimal Transport and Monge-Ampère

$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r} \right]$$

subject to:

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Optimality conditions

3. Optimal Transport and Monge-Ampère

$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r} \right]$$

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Optimality conditions

$$\frac{\partial \mathcal{L}}{\partial T} = 0 \quad \Rightarrow \quad \mathbf{r} = \nabla \Psi(T(\mathbf{r}))$$

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Legendre-Fenchel dual

3. Optimal Transport and Monge-Ampère

$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r} \right]$$

subject to:

$$\int g(\mathbf{q}) \bar{\rho} d\mathbf{q} = \int g(T(\mathbf{r})) \rho(\mathbf{r}) d\mathbf{r} \quad \forall g$$

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Legendre-Fenchel dual

Insert into constraint:

$$\bar{\rho} \int g(\nabla \Phi(\mathbf{r})) |\mathrm{D}^2 \Phi(\mathbf{r})| d\mathbf{r} = \int g(\nabla \Phi(\mathbf{r})) \rho(\mathbf{r}) d\mathbf{r}$$

3. Optimal Transport and Monge-Ampère

$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r} \right]$$

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Insert into constraint:

$$\bar{\rho} \int g(\nabla \Phi(\mathbf{r})) |\mathrm{D}^2 \Phi(\mathbf{r})| d\mathbf{r} \ = \ \int g(\nabla \Phi(\mathbf{r})) \rho(\mathbf{r}) d\mathbf{r}$$

Pointwise:

$$\bar{\rho} \det D^2 \Phi \ = \ \rho(\mathbf{r})$$

$$T(\mathbf{r}) \ = \ \nabla \Phi(\mathbf{r}), \quad \text{where:}$$

$$\Phi(\mathbf{r}) \ = \ \Psi^*(\mathbf{r}) = \inf_{\mathbf{q}} [\mathbf{q} \cdot \mathbf{r} - \Psi(\mathbf{q})]$$

Legendre-Fenchel dual

3. Optimal Transport and Monge-Ampère

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Legendre-Fenchel dual

Insert into constraint:

$$\bar{\rho} \int g(\nabla \Phi(\mathbf{r})) |\mathrm{D}^2 \Phi(\mathbf{r})| d\mathbf{r} = \int g(\nabla \Phi(\mathbf{r})) \rho(\mathbf{r}) d\mathbf{r}$$

Pointwise:

$$\bar{\rho} \det D^2 \Phi = \rho(\mathbf{r})$$

Monge-Ampère equation:

$$\bar{\rho} \mathbf{\Delta} \Phi = \rho$$

1. Newton

$$F = -\mathcal{G}\frac{m_1 m_2}{|\mathbf{r}_2 - \mathbf{r}_1|^2}$$

$$\left\{\begin{array}{lcl} F & = & \nabla\phi \\ \Delta\phi & = & 4\pi\mathcal{G}(\rho - \bar{\rho}) \end{array}\right.$$

2. Brenier-Monge-Ampère

$$\left\{\begin{array}{lcl} F & = & \nabla\Phi \\ \Delta\!\!\!\Delta\Phi & = & \frac{\rho}{\bar{\rho}} \\ \Phi & = & \frac{\phi}{4\pi\mathcal{G}\bar{\rho}} + \frac{|\mathbf{r}|^2}{2} \end{array}\right.$$

3. Optimal Transport

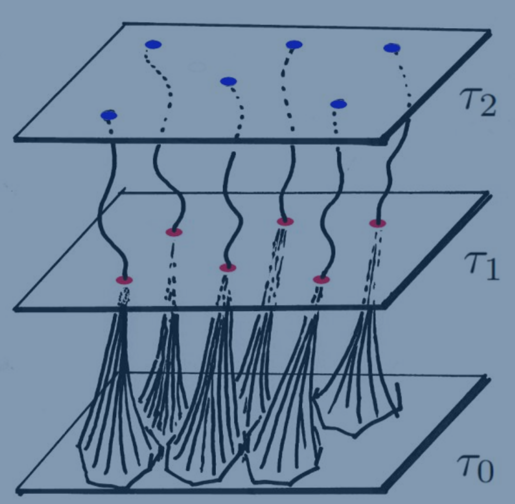
$$T \quad = \quad \nabla\Phi$$

$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r}\right]$$

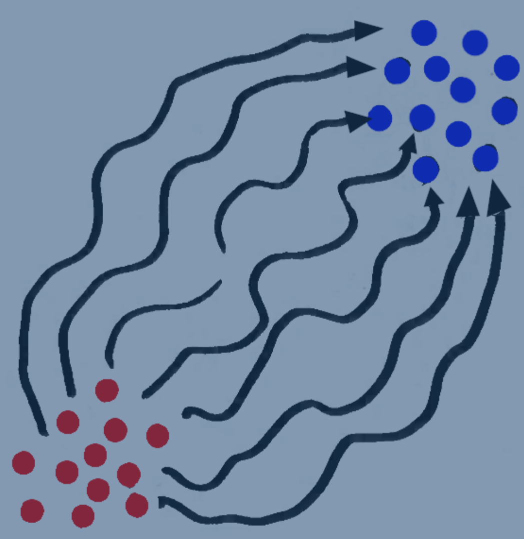
subject to:

$$\int_B \bar{\rho} d\mathbf{q} = \int_{T^{-1}(B)} \rho(\mathbf{r}) d\mathbf{r} \quad \forall B$$

6. The Path Bundle Method

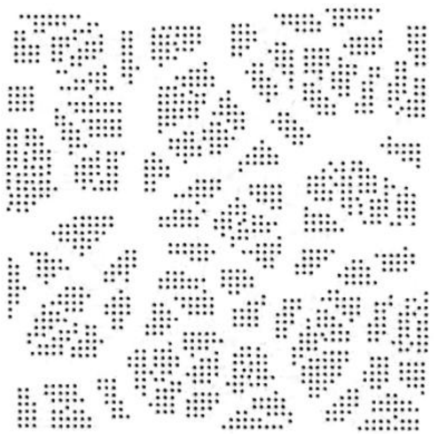


5. Large Deviations Pple.

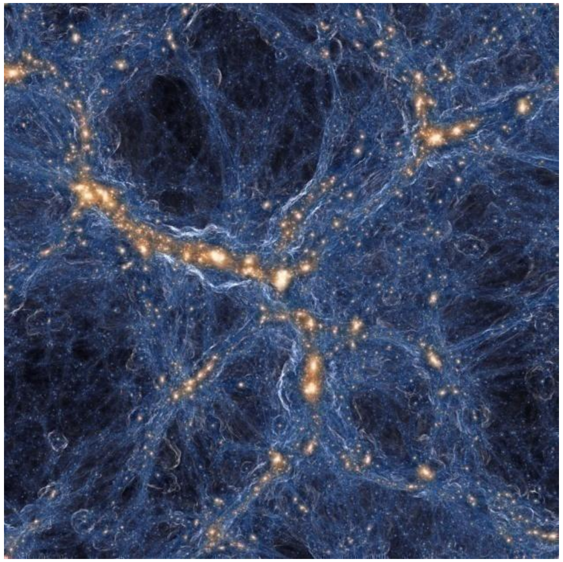


4. Discrete Optimal Transp.

$$\inf_{\sigma \in S_N} \left[\left|\mathbf{r}_i - \mathbf{q}_{\sigma(i)}\right|^2\right]$$



4. Discrete Optimal Transport

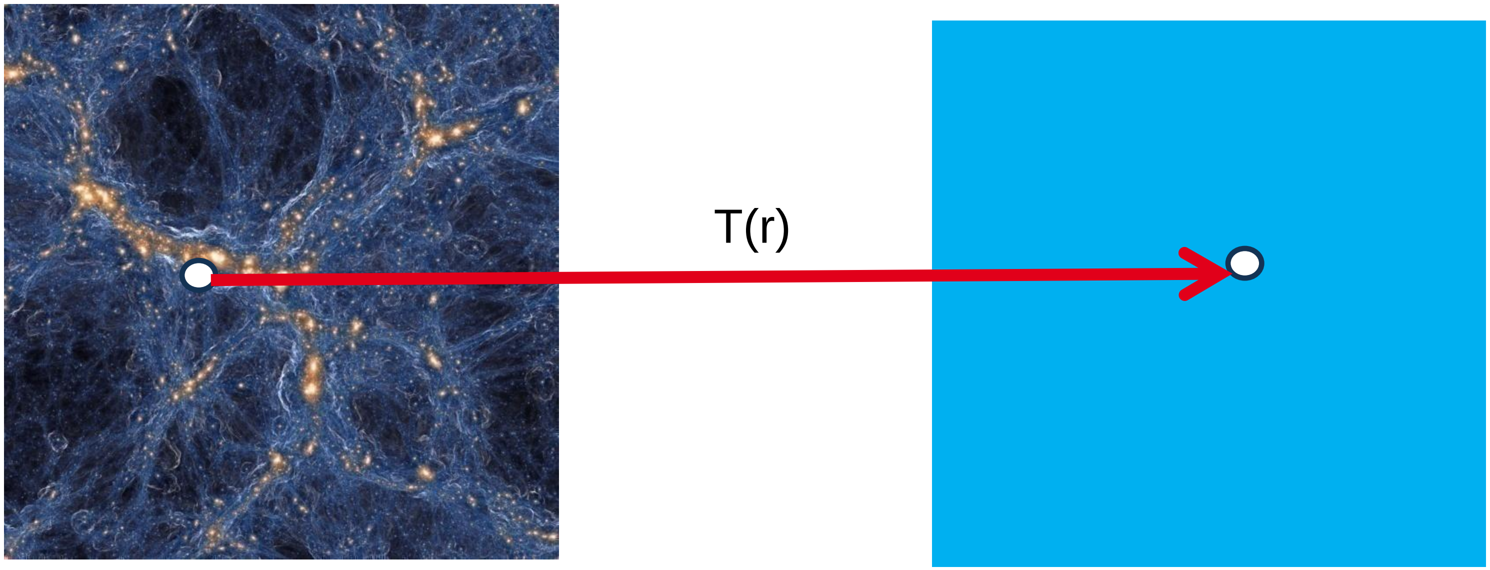


ρ

$\bar{\rho}$

$$
\begin{cases}
F & = & \nabla \Phi \\
\Delta \Phi & = & \frac{\rho}{\bar{\rho}} \\
\Phi & = & \frac{\phi}{4\pi \mathcal{G} \bar{\rho}} + \frac{|\mathbf{r}|^2}{2}
\end{cases}
$$

4. Discrete Optimal Transport

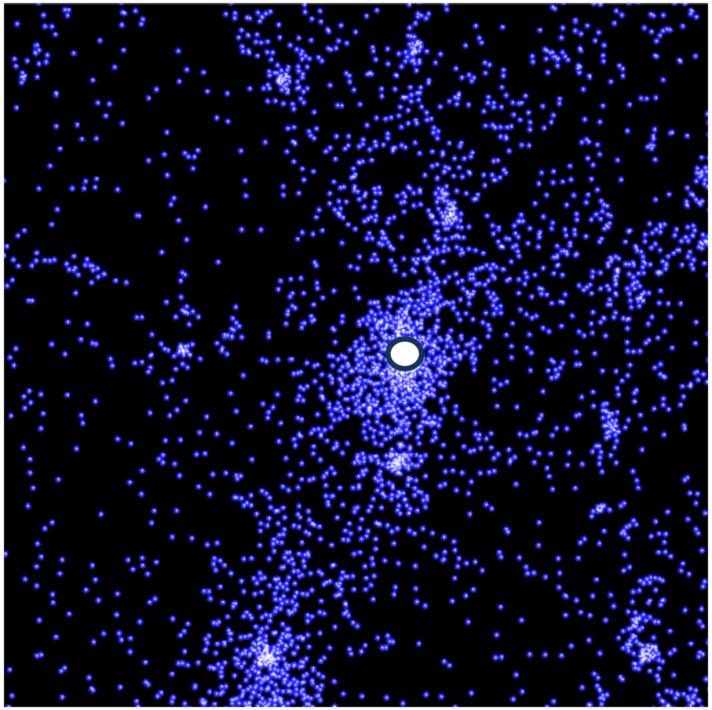


ρ $\qquad\qquad$ $\bar{\rho}$

$$\begin{cases} F &= \nabla \Phi \\ \Delta \Phi &= \frac{\rho}{\bar{\rho}} \\ \Phi &= \frac{\phi}{4\pi \mathcal{G} \bar{\rho}} + \frac{|\mathbf{r}|^2}{2} \end{cases}$$

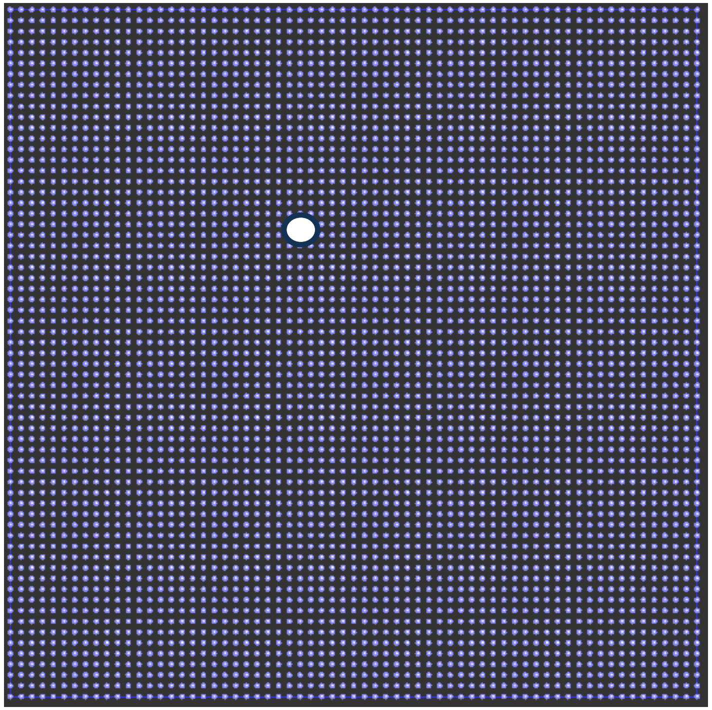
$$F = \frac{1}{4\pi \mathcal{G} \bar{\rho}} (\mathbf{r} - T(\mathbf{r}))$$

4. Discrete Optimal Transport



ρ

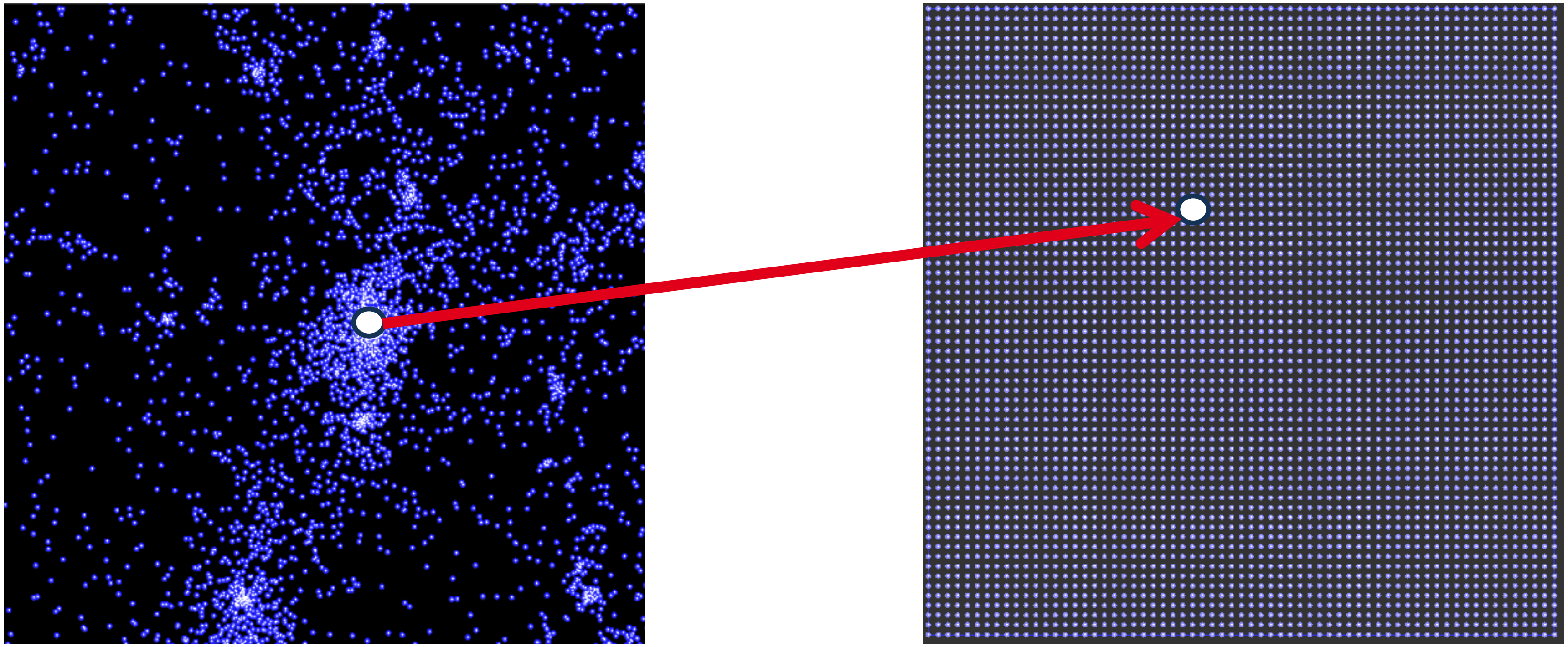
N points $\mathbf{r}_i$



$\bar{\rho}$

N points $\mathbf{q}_i$

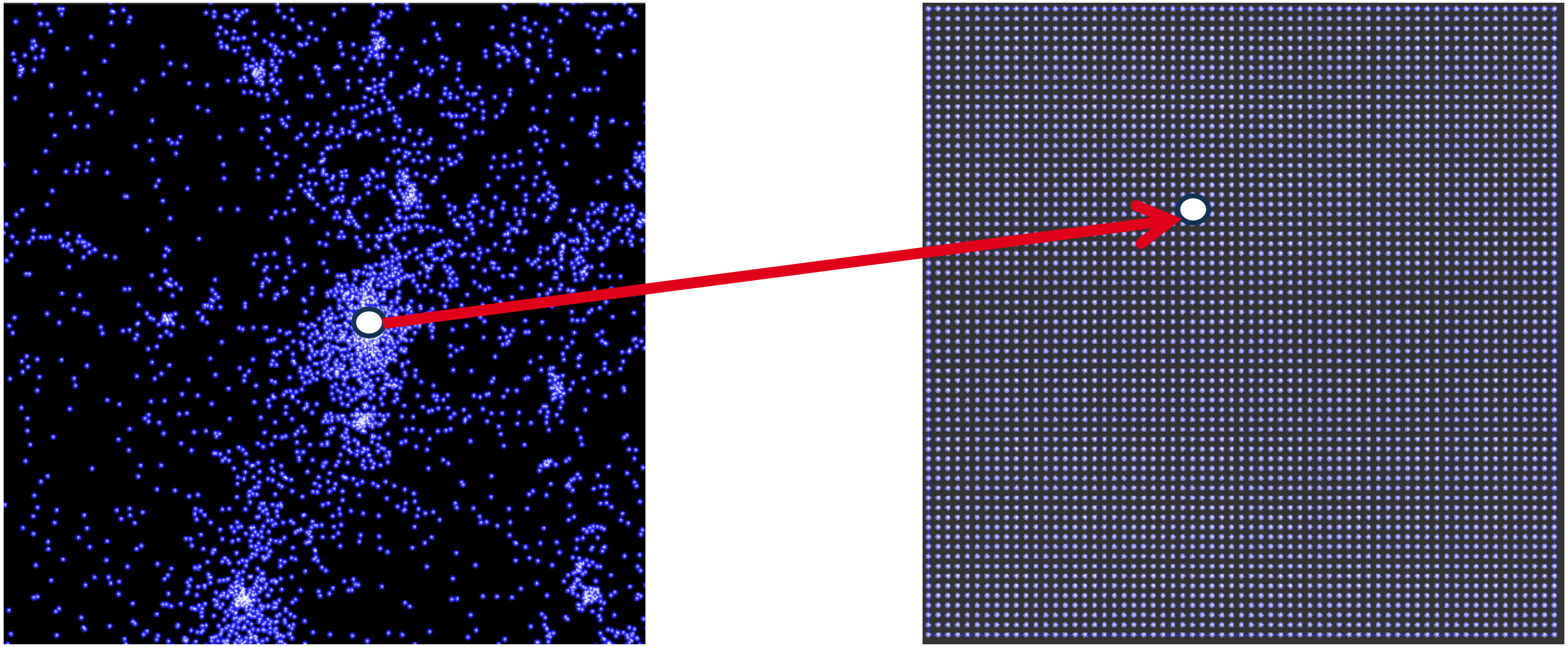
4. Discrete Optimal Transport



ρ $\bar{\rho}$

$$T(\mathbf{r}_i) = \mathbf{q}_{\sigma(i)}$$

4. Discrete Optimal Transport



ρ $\bar{\rho}$

$$T(\mathbf{r}_i) = \mathbf{q}_{\sigma(i)}$$

σ :*The permutation that minimizes* $\left[\left|\mathbf{r}_i - \mathbf{q}_{\sigma(i)}\right|^2\right]$

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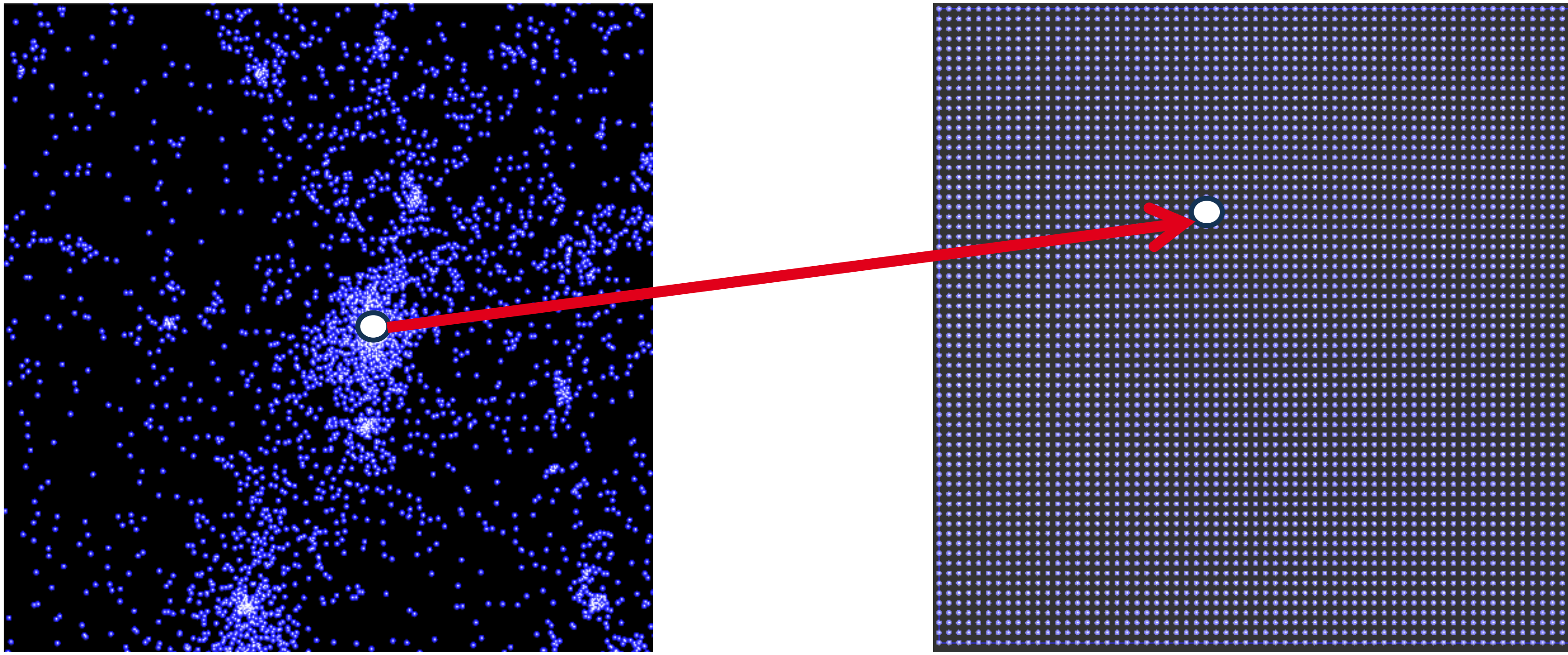

ρ

$T(\mathbf{r}_i) = \mathbf{q}_{\sigma(i)}$

$\bar{\rho}$

$$F = \frac{1}{4\pi\mathcal{G}\bar{\rho}}(\mathbf{r} - T(\mathbf{r}))$$

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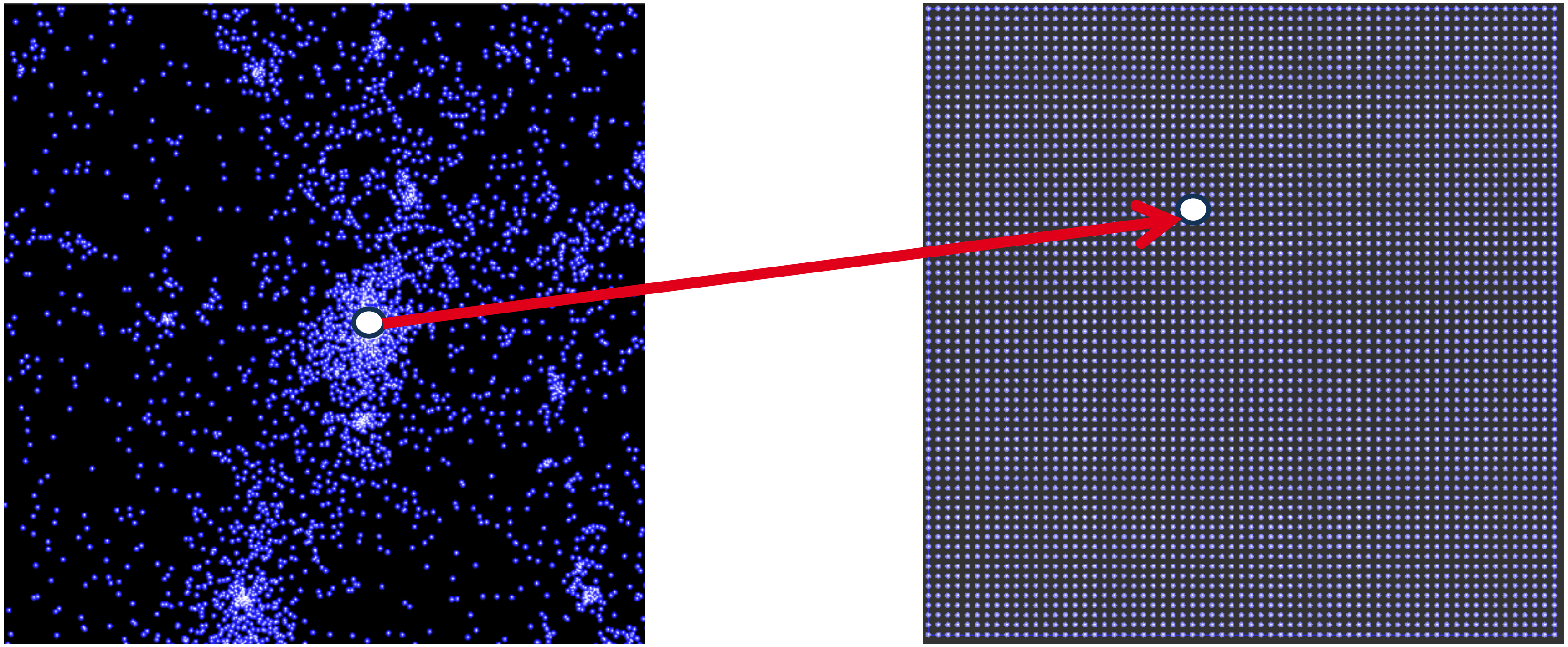

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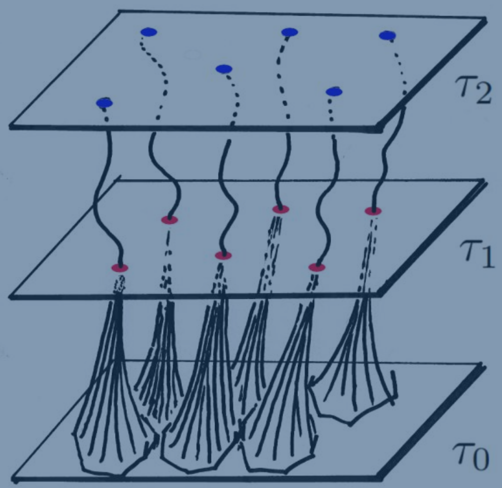
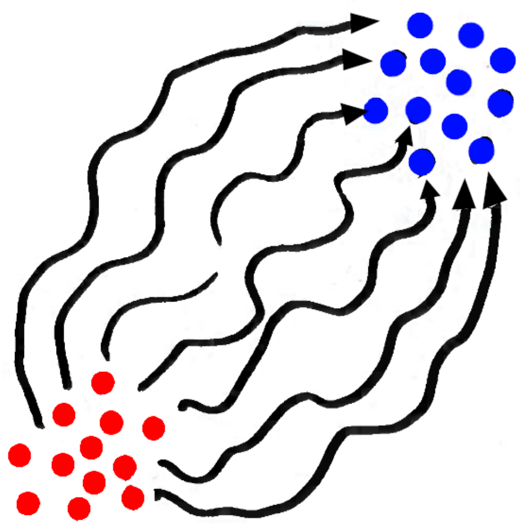
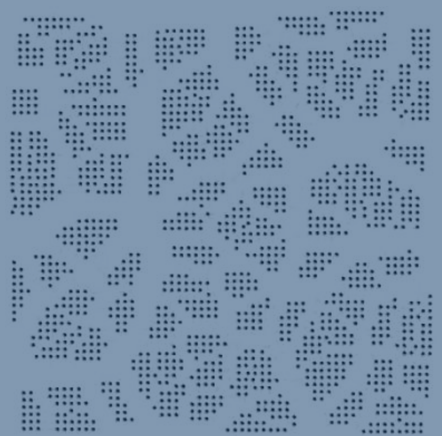


5. Large Deviations Pple.



4. Discrete Optimal Transp.

$$\inf_{\sigma \in S_N} \left[\left| \mathbf{r}_i - \mathbf{q}_{\sigma(i)} \right|^2 \right]$$



5. Large Deviation Principle

$$F_i = \frac{1}{4\pi\mathcal{G}\bar{\rho}}(\mathbf{r}_i - \mathbf{q}_{\sigma(i)})$$

σ :*The permutation that minimizes* $\left[\left|\mathbf{r}_i - \mathbf{q}_{\sigma(i)}\right|^2\right]$

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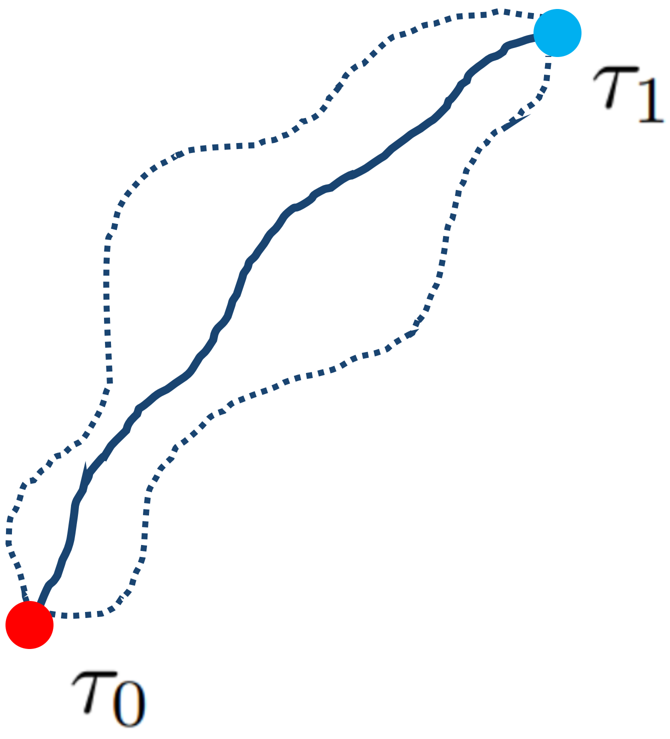
Why ?

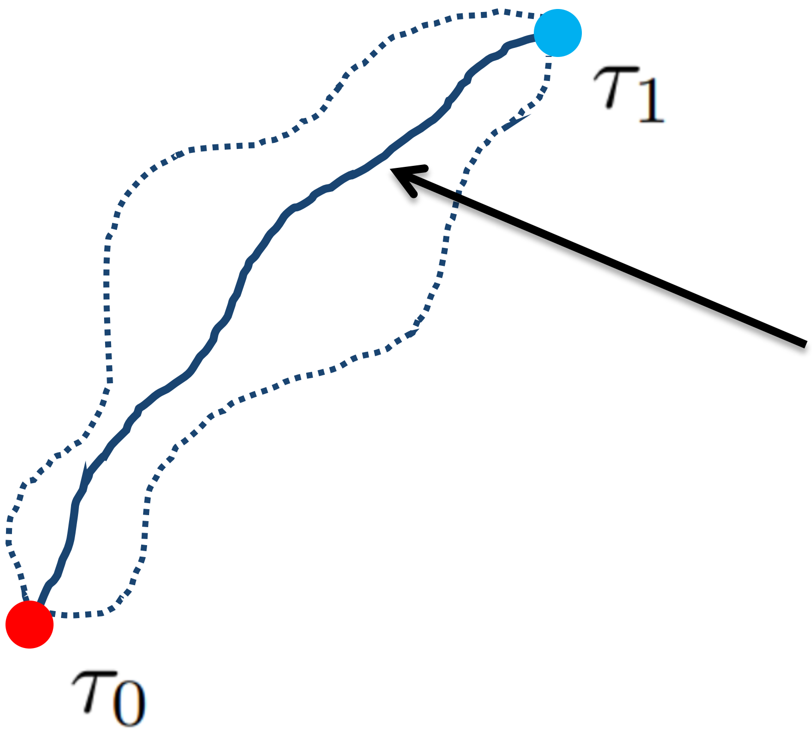
Can we *deduce* this formula from something else ?

5. Large Deviation Principle



Idea has similarities with ***least action***

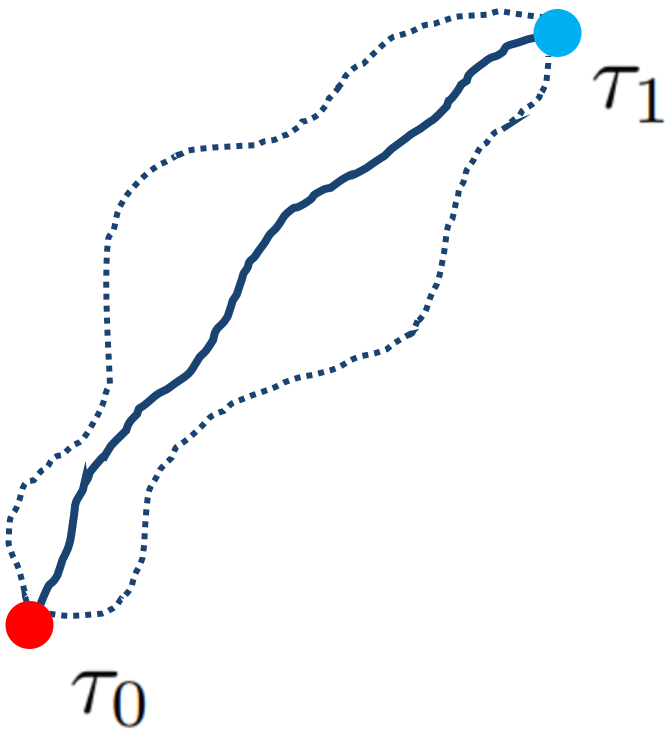
5. Large Deviation Principle



Idea has similarities with ***least action***

Extremize action between ***fixed*** initial and final conditions.

5. Large Deviation Principle

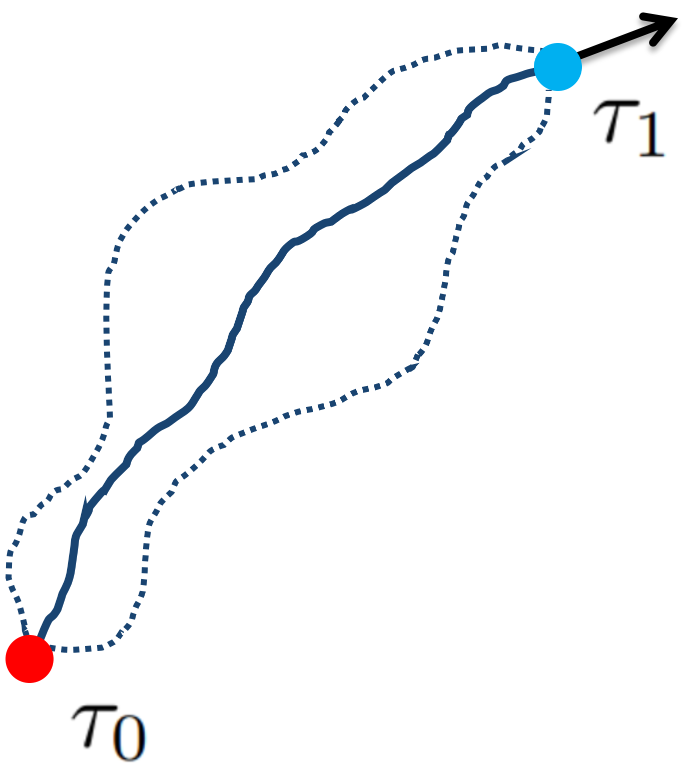


Idea has similarities with ***least action***

Extremize action between ***fixed*** initial and final conditions.

Deduce law of motion (differential relation)

5. Large Deviation Principle



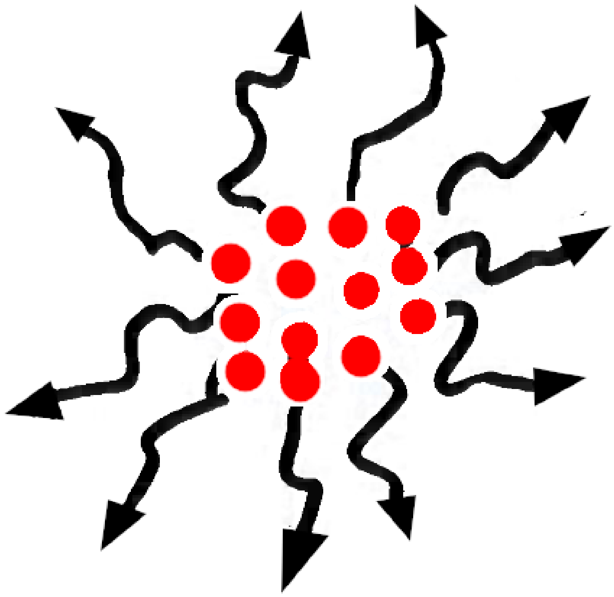
Idea has similarities with ***least action***

Extremize action between ***fixed*** initial and final conditions.

Deduce law of motion (differential relation)

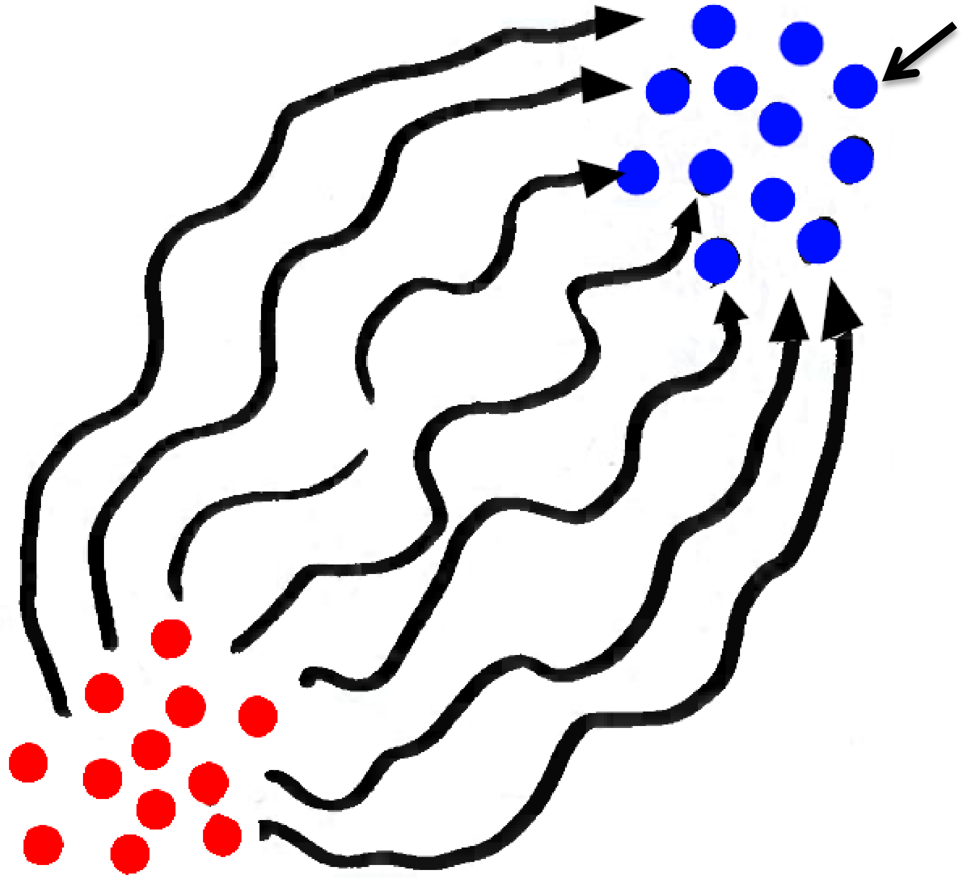
Extrapolate it

5. Large Deviation Principle



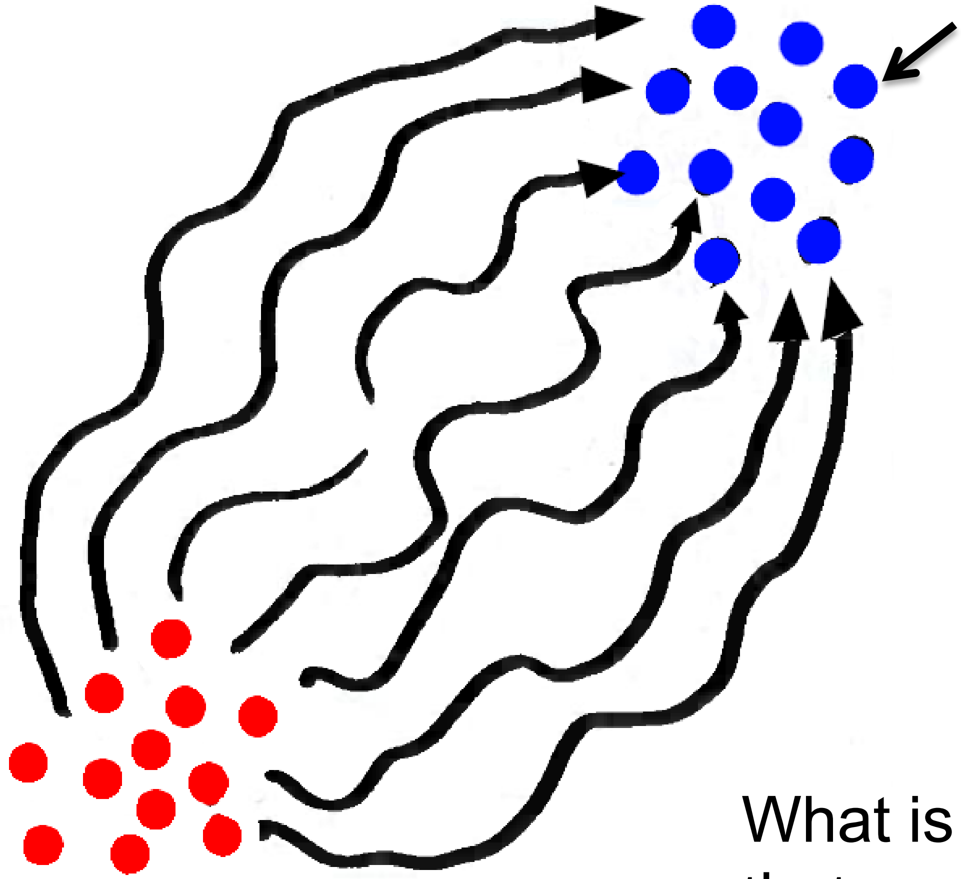
M ***indistinguishable*** particles
Independent Brownian motion
No interaction

5. Large Deviation Principle



We suppose that we observe them here after T seconds

5. Large Deviation Principle

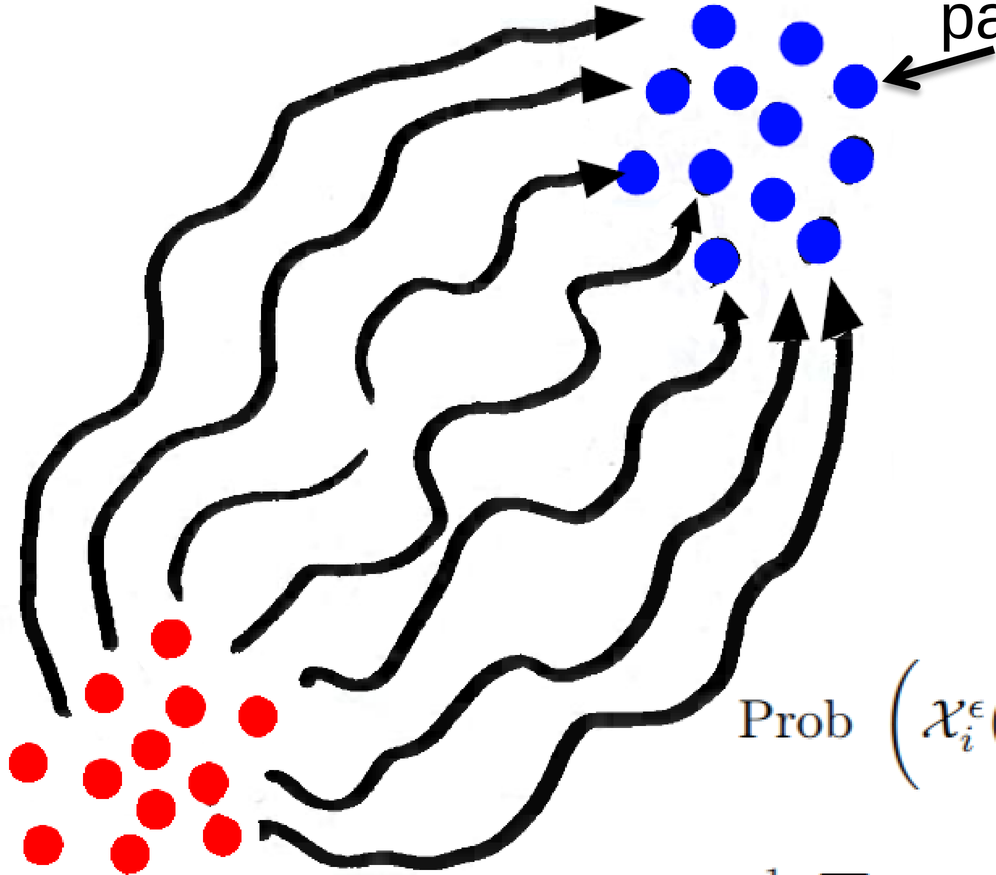


We suppose that we observe them here after T seconds

What is the “most probable” motion that accounts for the observation ?

5. Large Deviation Principle

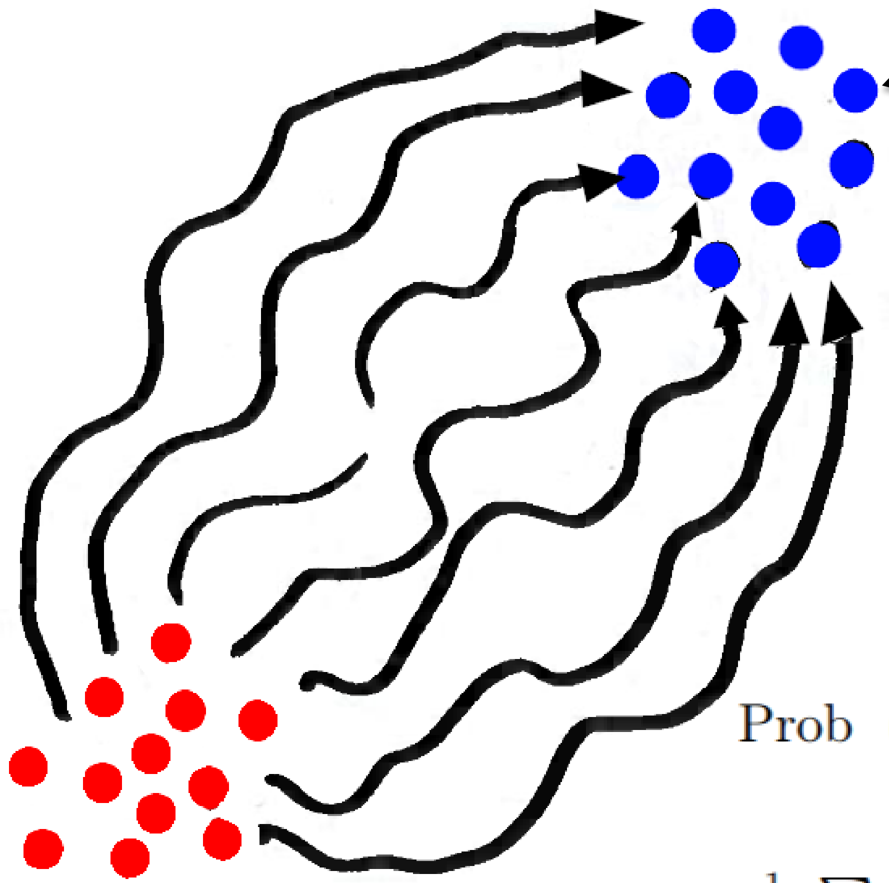
Probability of observing the particles here after T seconds:



$$\text{Prob}\left(\mathcal{X}_i^\epsilon(T) \underset{perm}{\approx} Y\right) \approx$$

$$\frac{1}{M!}\sum_{\sigma\in S_M} \exp\left[\frac{-\sum_i |Y_{\sigma(i)} - X_i^0|^2}{2\epsilon T}\right](2\pi\epsilon T)^{-\frac{3M}{2}}$$

5. Large Deviation Principle



Probability of observing the particles here after T seconds:

It’s a soft inf !

$$\mathrm{Prob}\left(\mathcal{X}_i^{\epsilon}(T) \underset{perm}{\approx} Y\right) \approx$$

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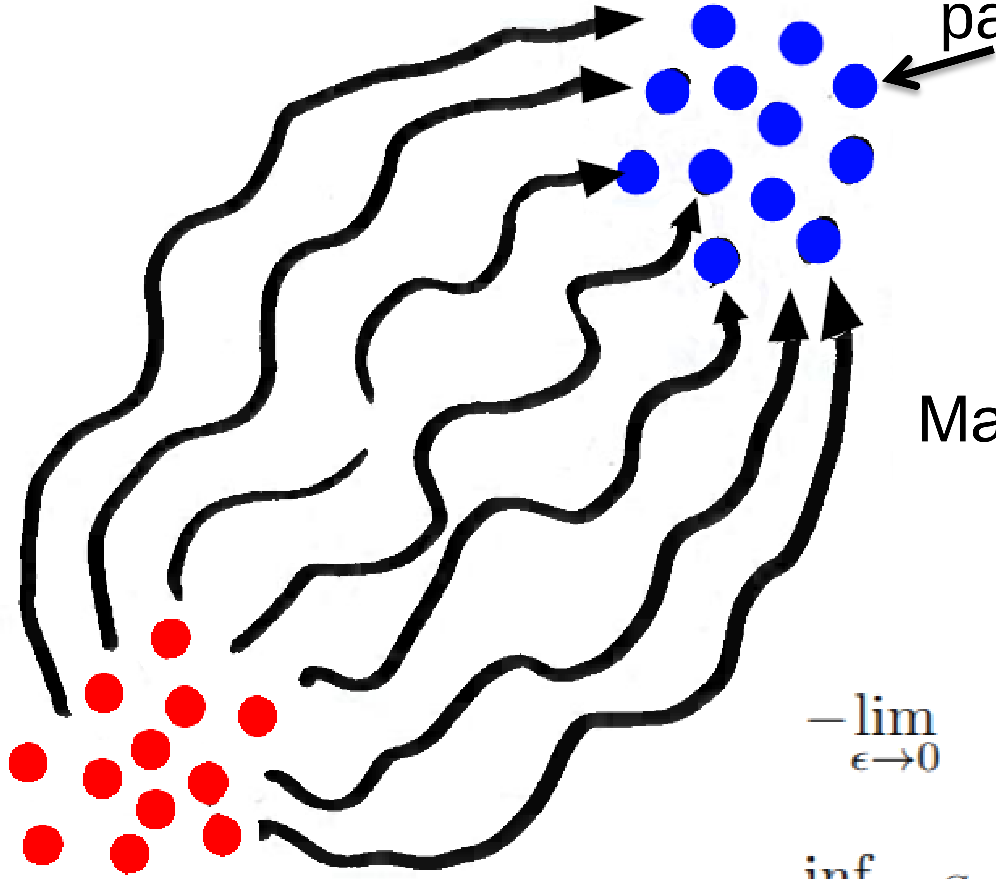
5. Large Deviation Principle



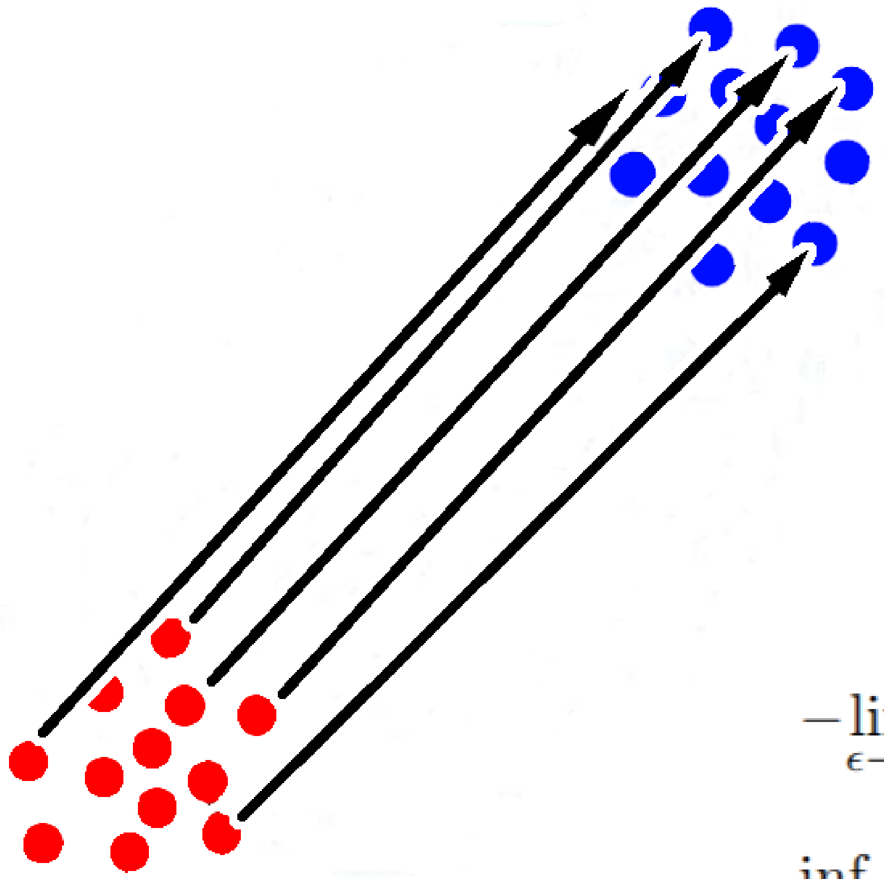
Probability of observing the particles here after T seconds:

Make “temperature” ϵ tend to 0:

$$-\lim_{\epsilon \to 0} \epsilon \log \text{Prob} \left[\mathcal{X}_i^\epsilon(T) \underset{perm}{\approx} Y \right] \approx$$

$$\inf_{\sigma \in S_N} \left[\frac{\sum_i |Y_{\sigma(i)} - X_i^0|^2}{2T} \right]$$

5. Large Deviation Principle



Trajectories become geodesics

$$-\lim_{\epsilon\to 0} \ \epsilon \log \mathrm{Prob} \left[\mathcal{X}_i^\epsilon(T) \underset{perm}{\approx} Y \right] \approx$$

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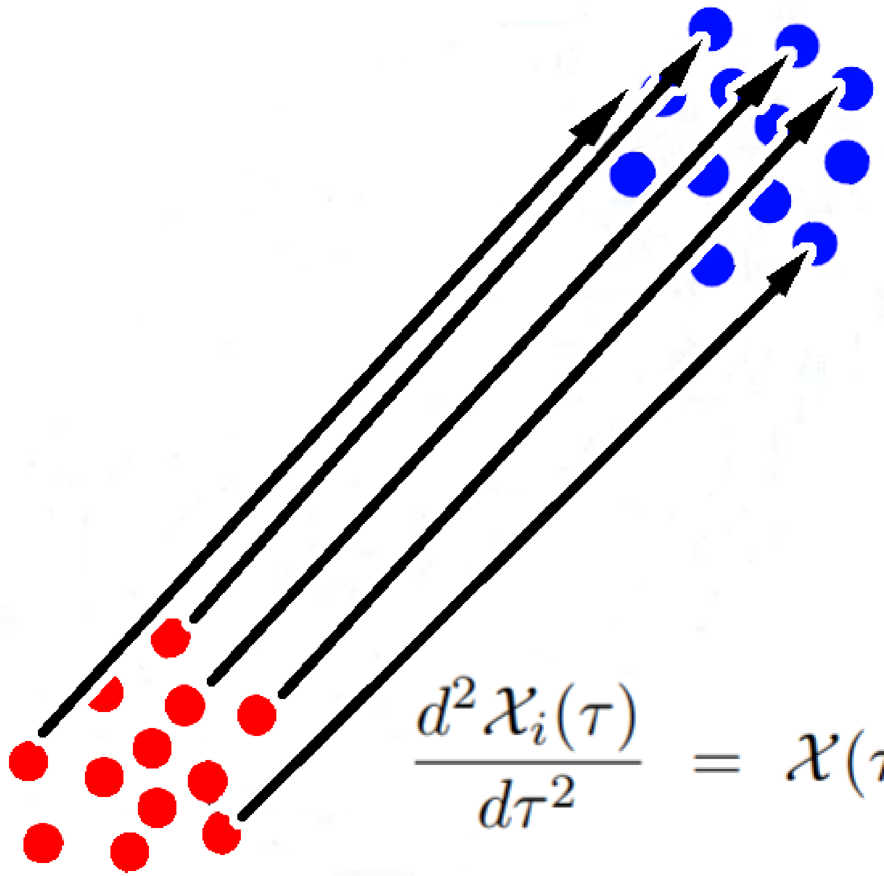
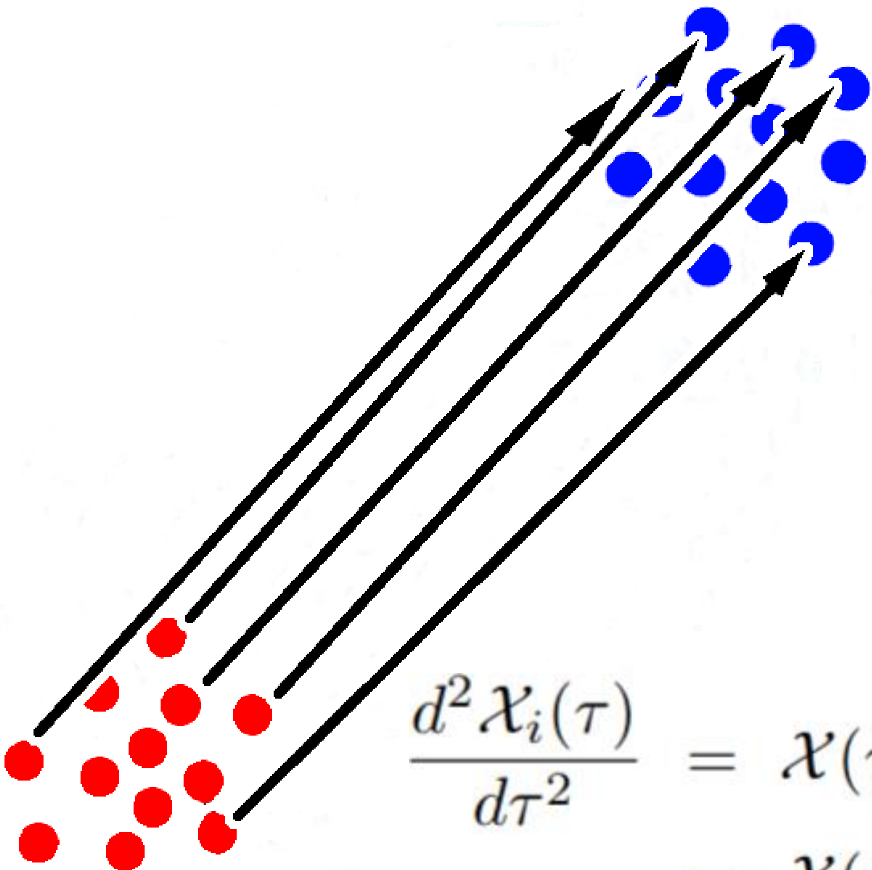


Along these geodesics:

$$\frac{d^2 \mathcal{X}_i(\tau)}{d\tau^2} = \mathcal{X}(\tau) - \mathbf{q}_{(\sigma|\mathcal{X}(\tau))(i)}$$

5. Large Deviation Principle



Along these geodesics:

$$\frac{d^2\mathcal{X}_i(\tau)}{d\tau^2} = \mathcal{X}(\tau) - \mathbf{q}_{(\sigma|\mathcal{X}(\tau))(i)}$$
$$= \mathcal{X}(\tau) - \nabla\Phi(\mathcal{X}(t)) = -\nabla\phi(\mathcal{X}(\tau))$$

1. Newton

$$F = -\mathcal{G}\frac{m_1 m_2}{|\mathbf{r}_2 - \mathbf{r}_1|^2}$$

$$\begin{cases} F & = & \nabla\phi \\ \Delta\phi & = & 4\pi\mathcal{G}(\rho - \bar{\rho}) \end{cases}$$

2. Brenier-Monge-Ampère

$$\begin{cases} F & = & \nabla\Phi \\ \boldsymbol{\Delta}\Phi & = & \frac{\rho}{\bar{\rho}} \\ \Phi & = & \frac{\phi}{4\pi\mathcal{G}\bar{\rho}} + \frac{|\mathbf{r}|^2}{2} \end{cases}$$

3. Optimal Transport

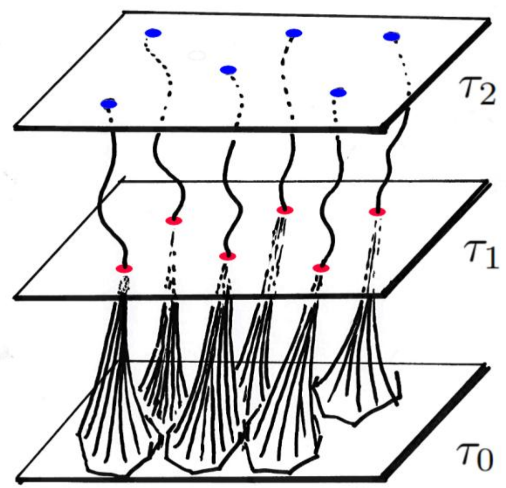
$$T = \nabla\Phi$$

$$\inf_T \left[\int_V |\mathbf{r} - T(\mathbf{r})|^2 \rho(\mathbf{r}) d\mathbf{r}\right]$$

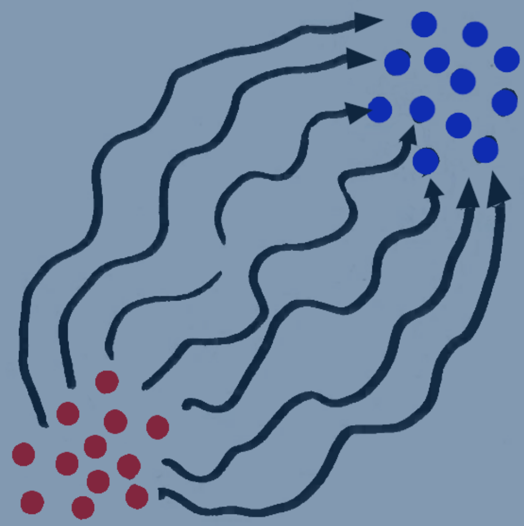
subject to:

$$\int_B \bar{\rho} d\mathbf{q} = \int_{T^{-1}(B)} \rho(\mathbf{r}) d\mathbf{r} \quad \forall B$$

6. The Path Bundle Method

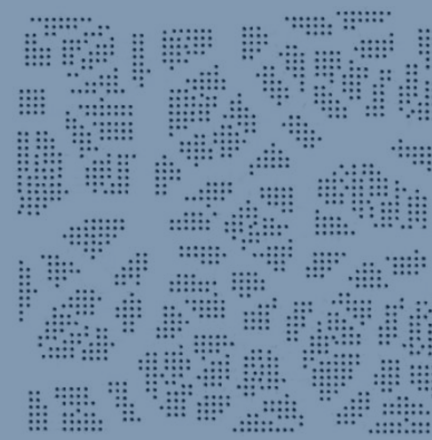


5. Large Deviations Pple.



4. Discrete Optimal Transp.

$$\inf_{\sigma \in S_N} \left[\left|\mathbf{r}_i - \mathbf{q}_{\sigma(i)}\right|^2\right]$$



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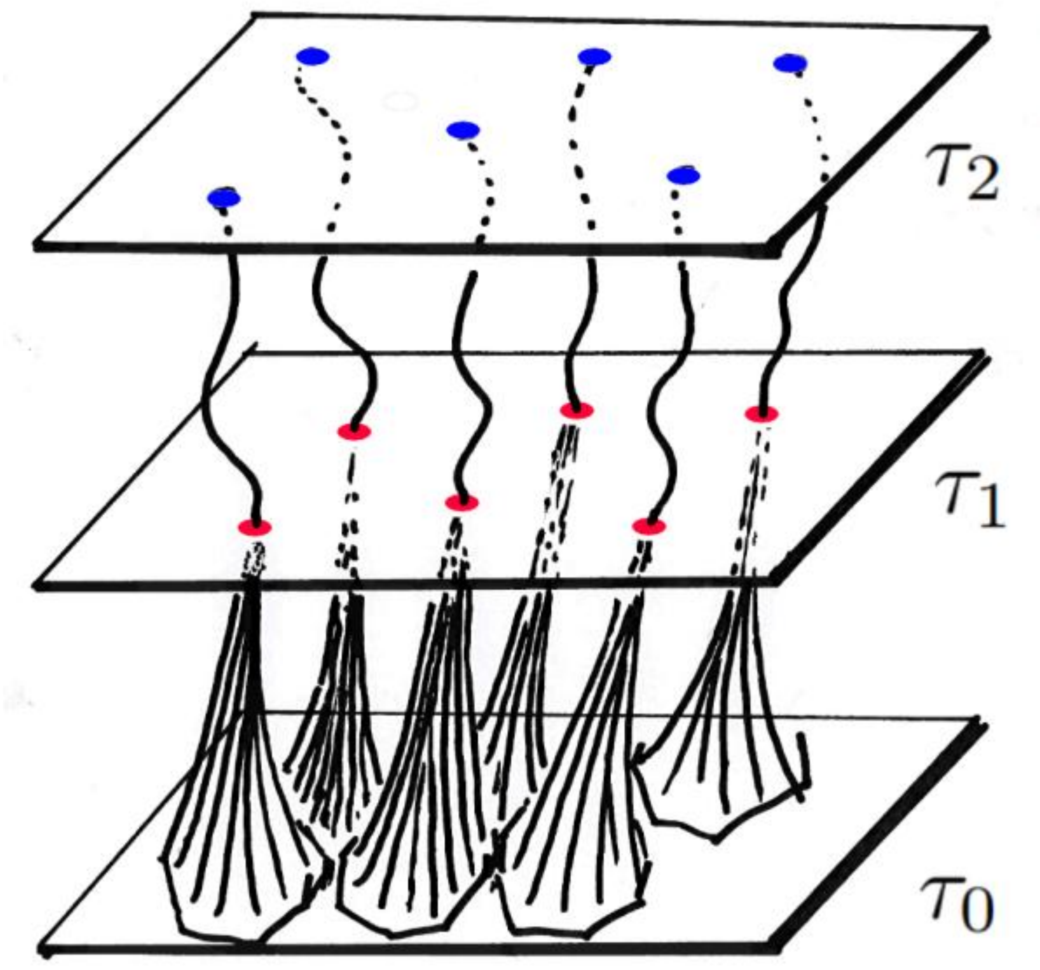
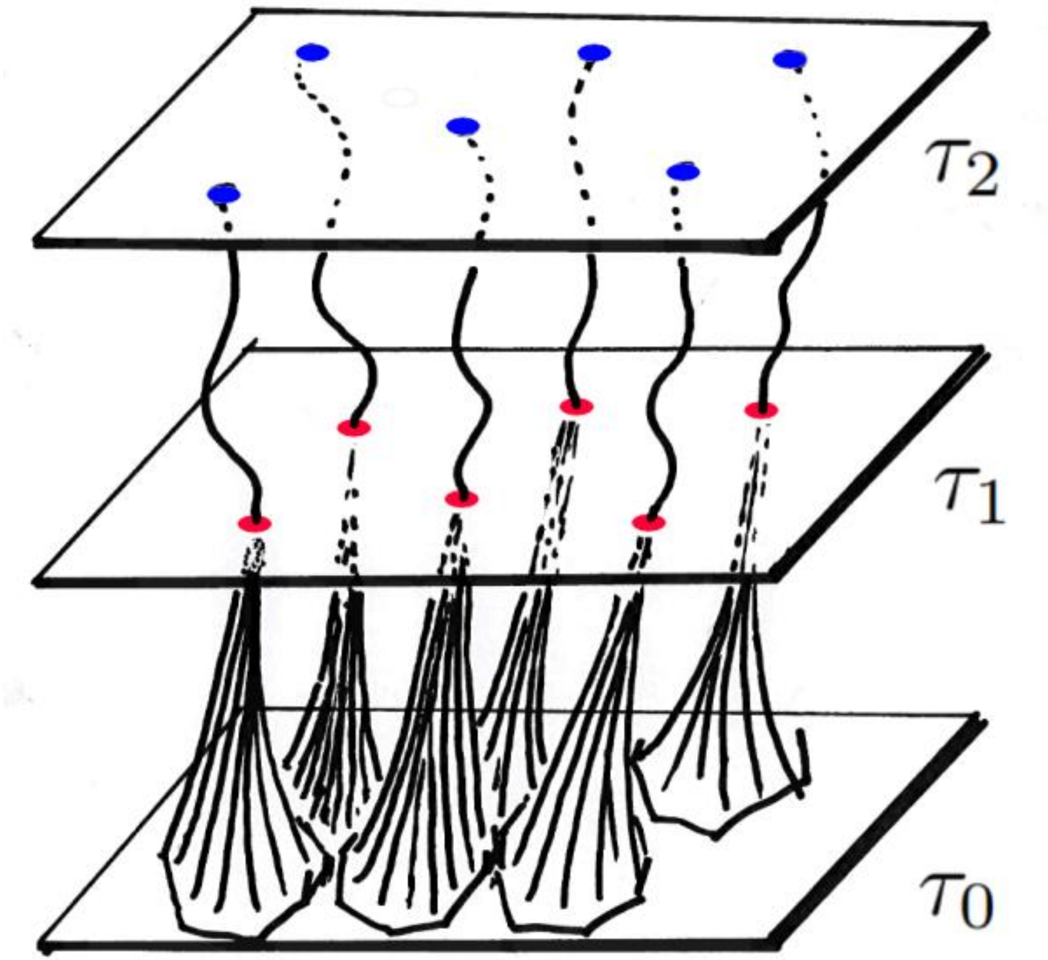


Initial condition (homogeneous)

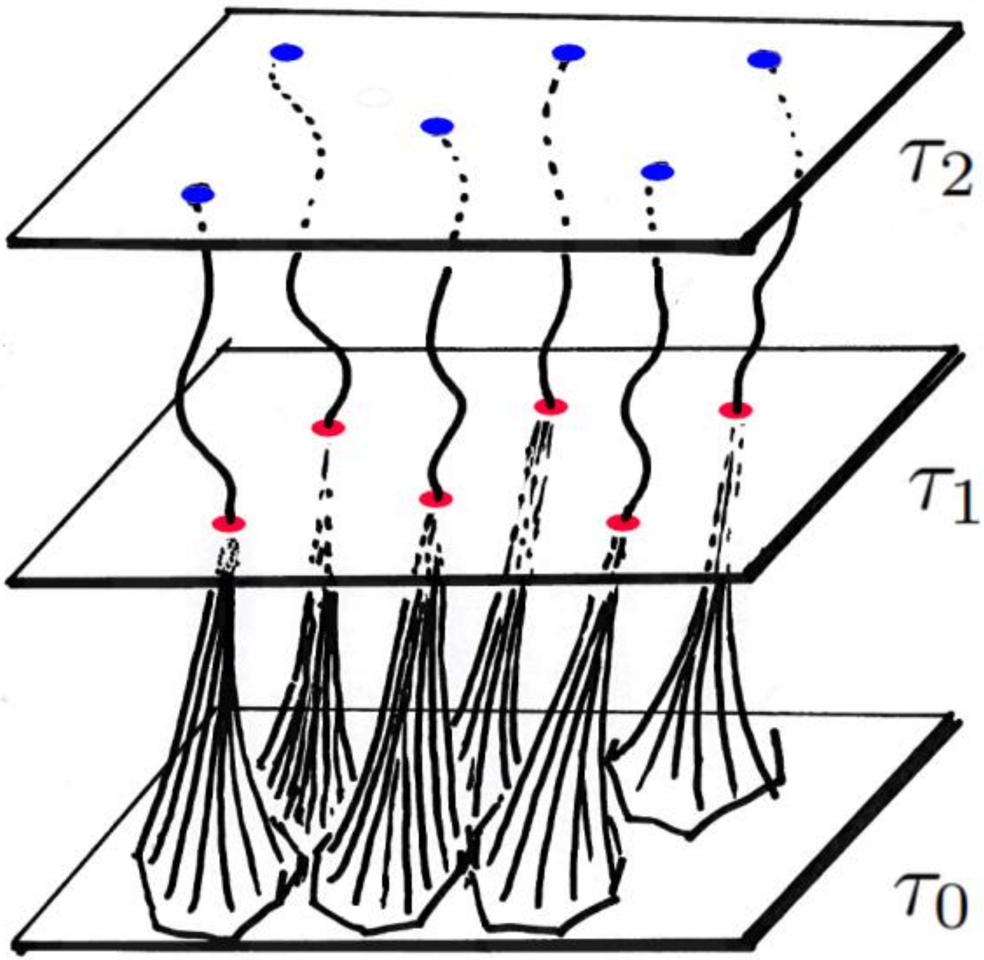
6. The Path Bundle Method



Structures formation

Initial condition (homogeneous)

6. The Path Bundle Method



Observation

Structures formation

Initial condition (homogeneous)

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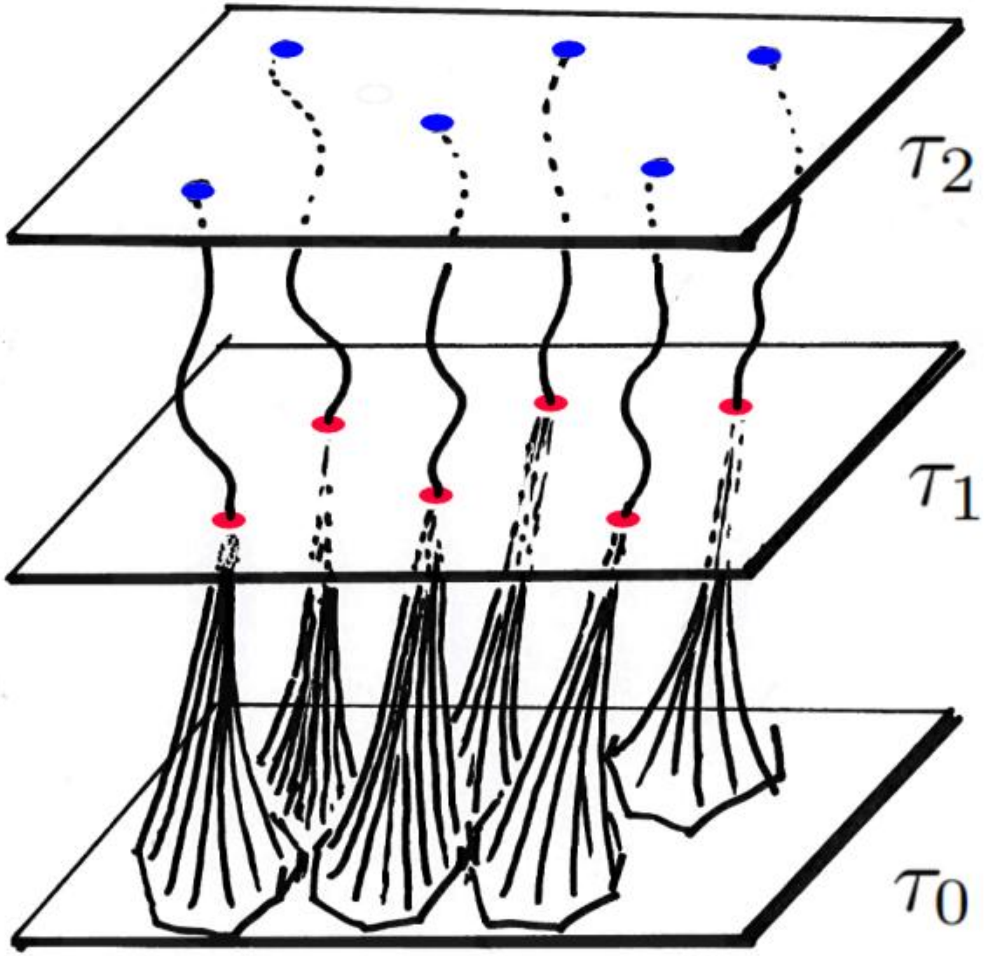
$$\frac{d^2\mathbf{r}_i(\tau)}{d\tau^2} = F_i(\tau)$$



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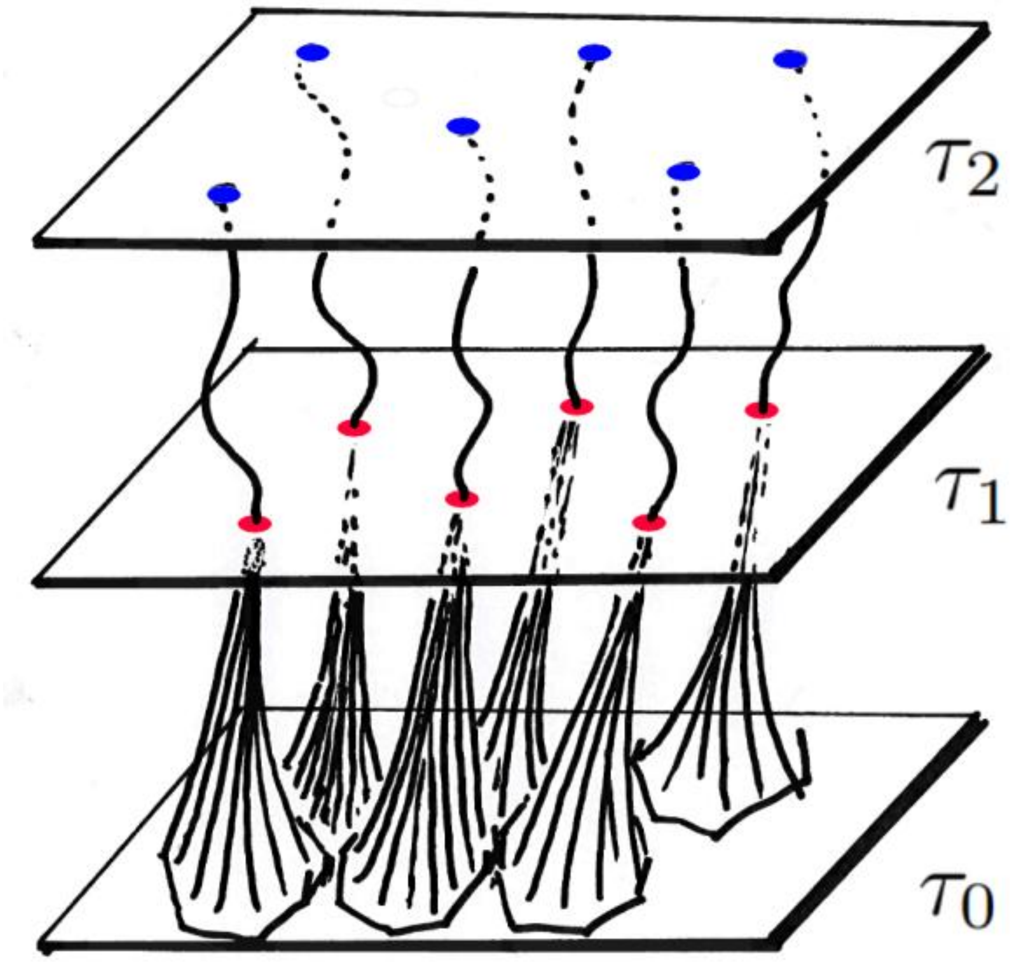

$$\frac{d^2\mathbf{r}_i(\tau)}{d\tau^2} = F_i(\tau)$$

$$\begin{aligned} F_i(\tau) &= -\nabla\phi(\tau) \\ &= \mathbf{r}_i - \nabla\Phi(\mathbf{r}_i, \tau) \end{aligned}$$

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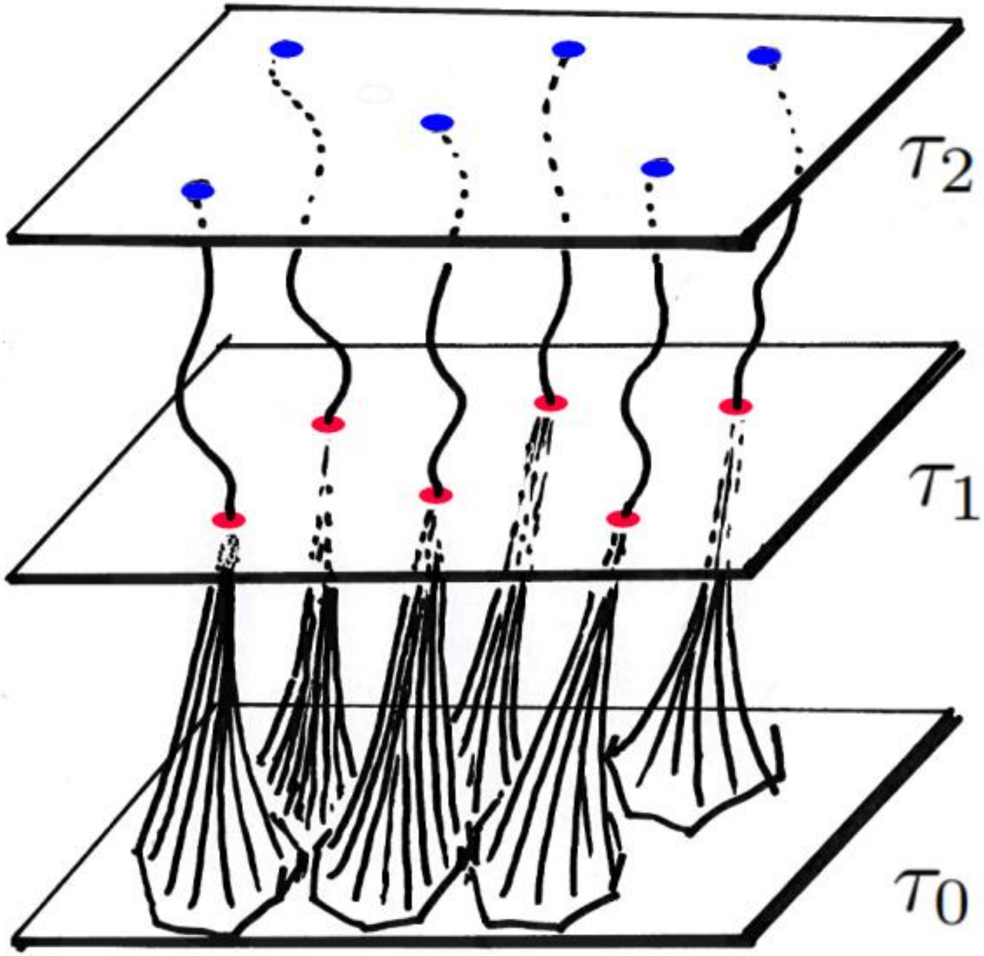

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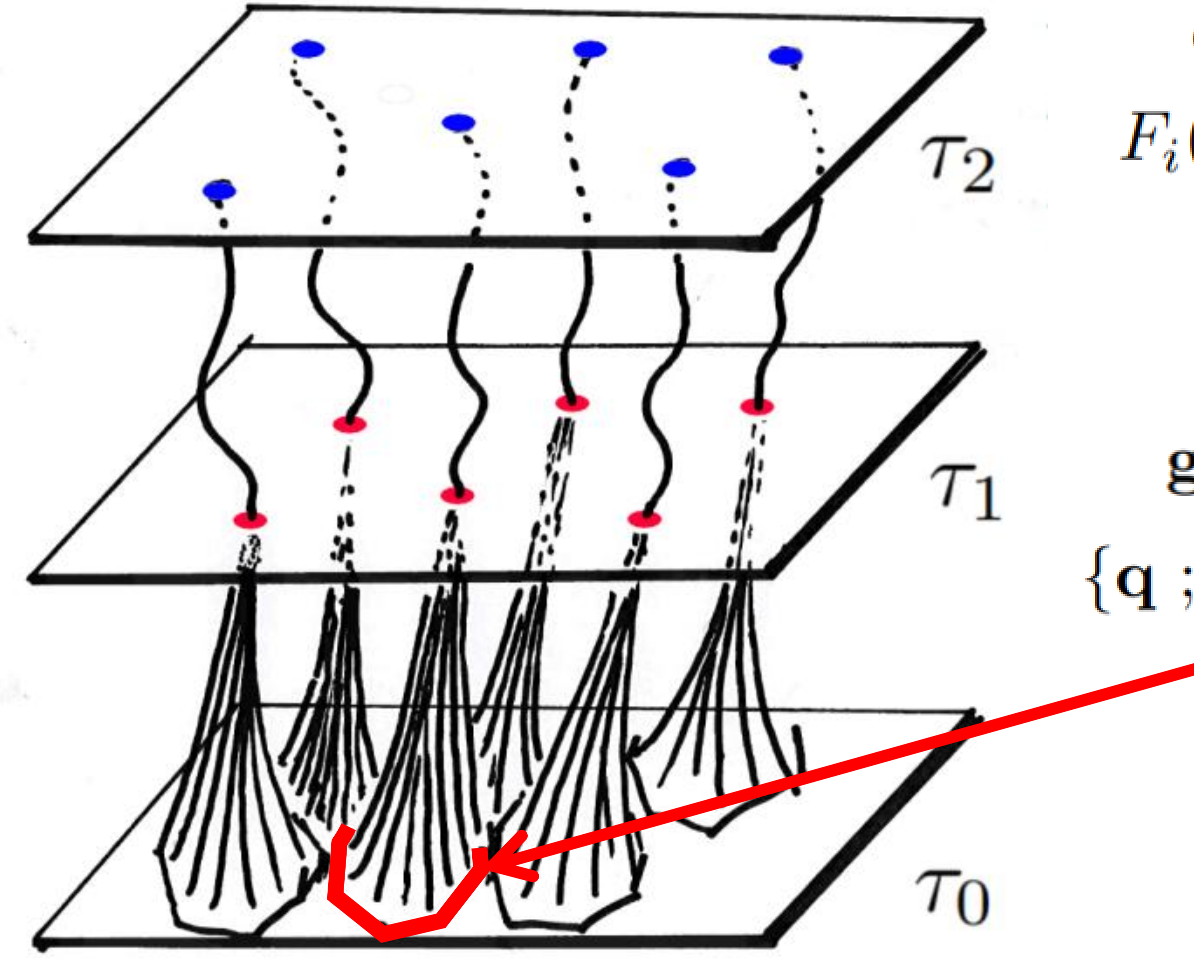

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$\mathbf{g}_i(\tau)$: barycenter of

$$\{\mathbf{q}\ ;\ |\mathbf{q} - \mathbf{r}_i|^2 - \phi_i \leq |\mathbf{q} - \mathbf{r}_j|^2 - \phi_j \quad \forall 1 \leq j \leq N\}$$

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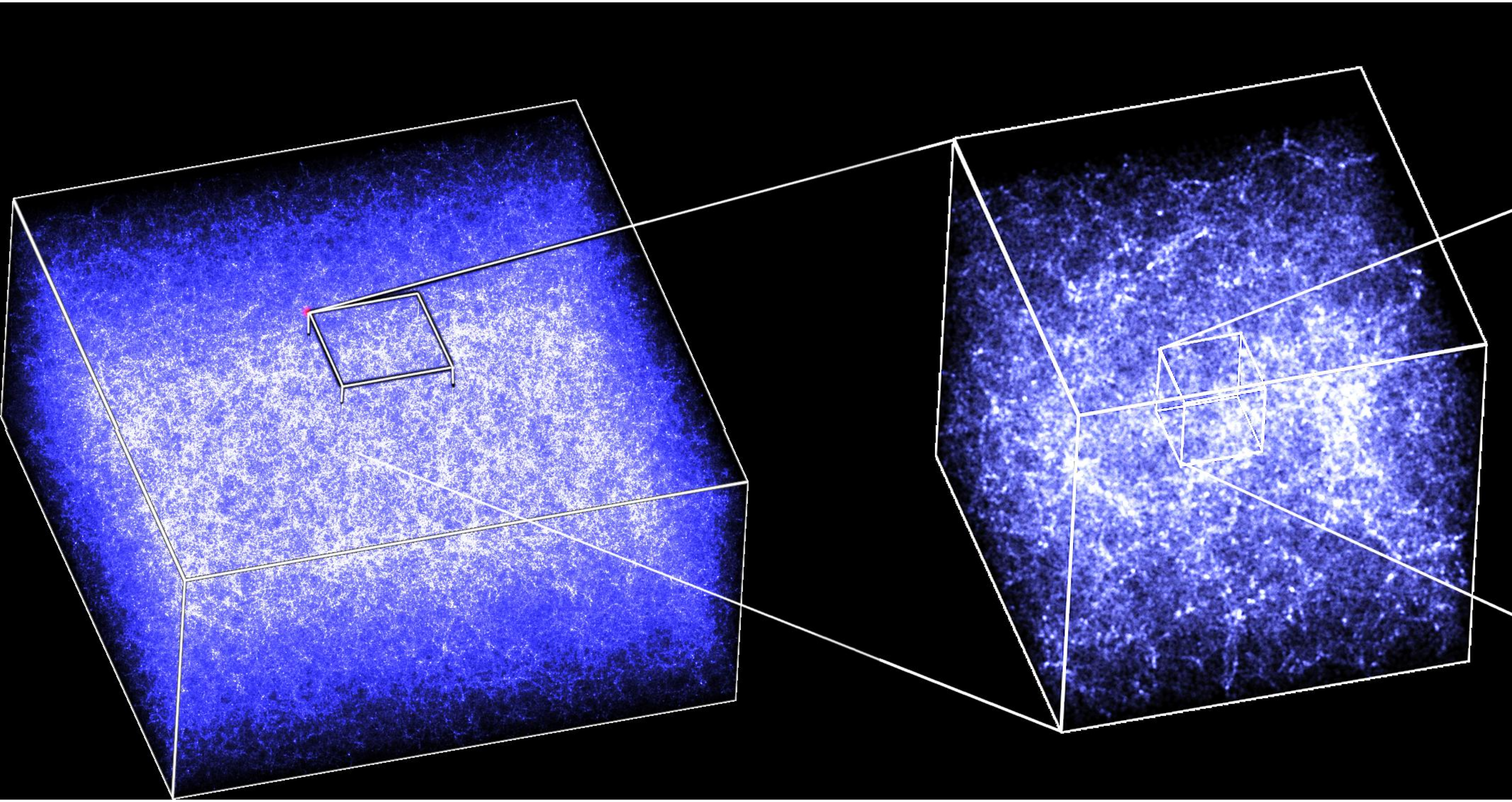

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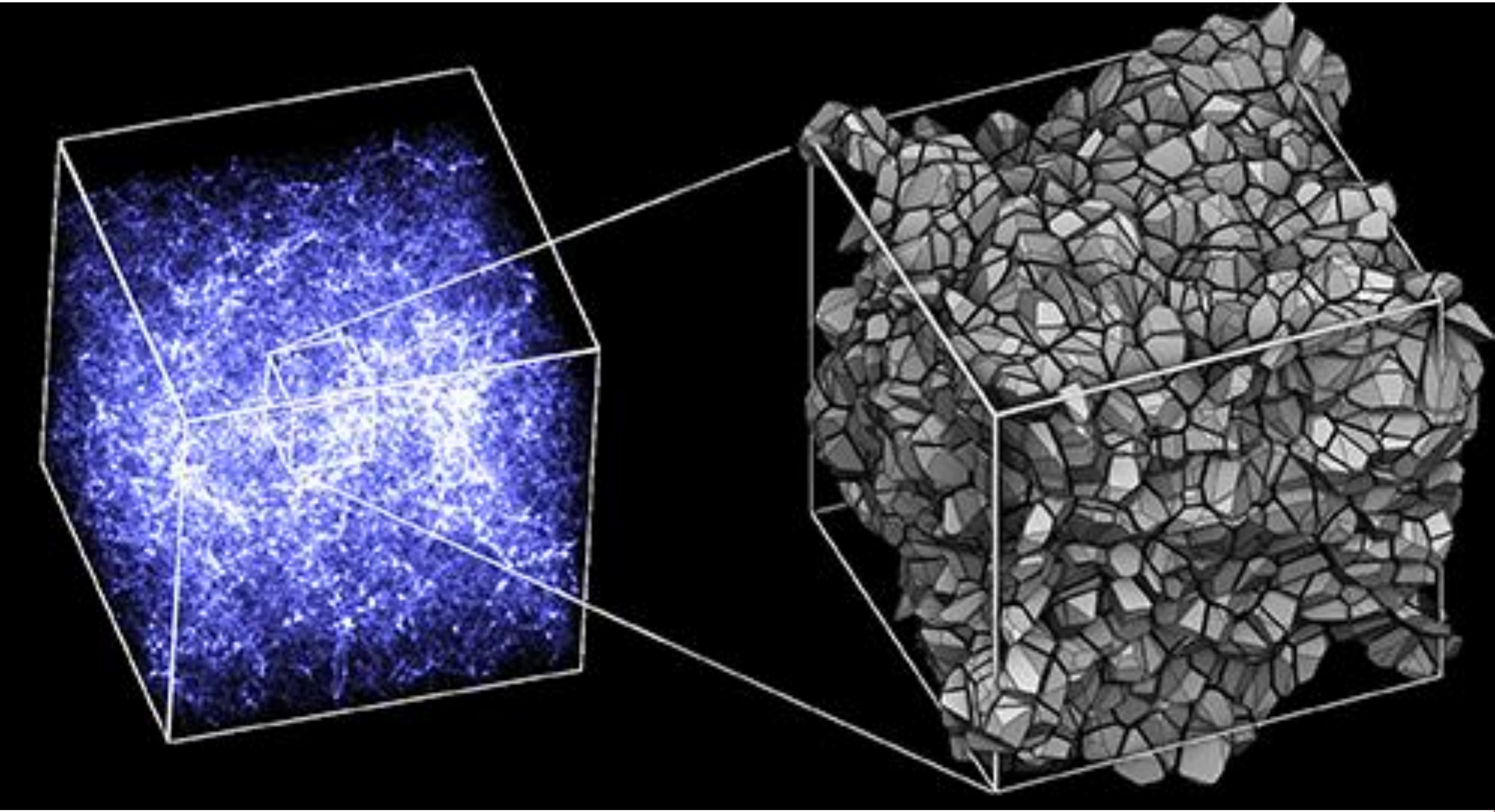
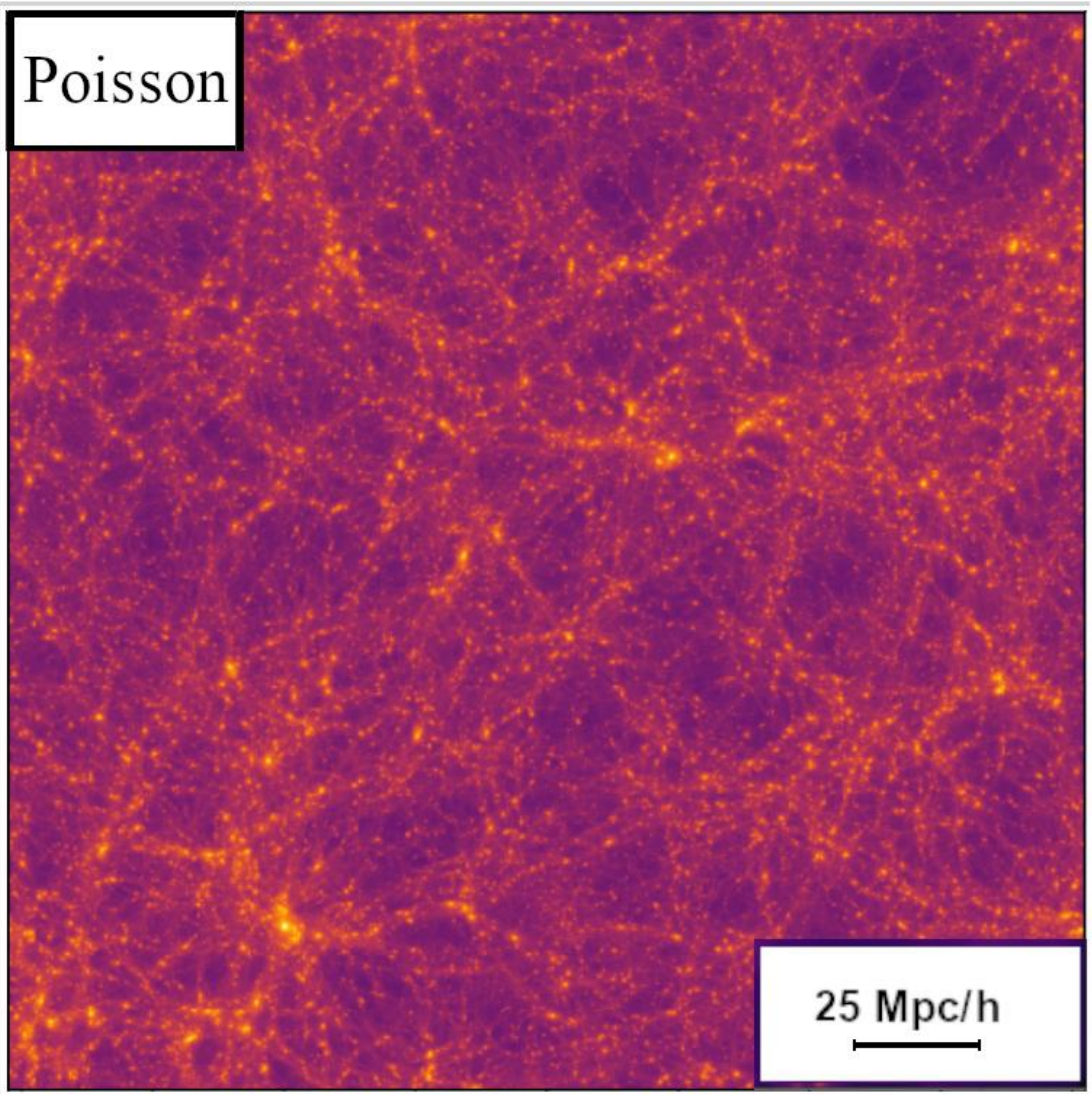


Results – Cosmological simulation

- 300 million particles
- 200 Mpc/h
- Λ-CDM initial conditions [Planck]
- Newton-Poisson and BMAG

Results – Simulation with 300 M cells



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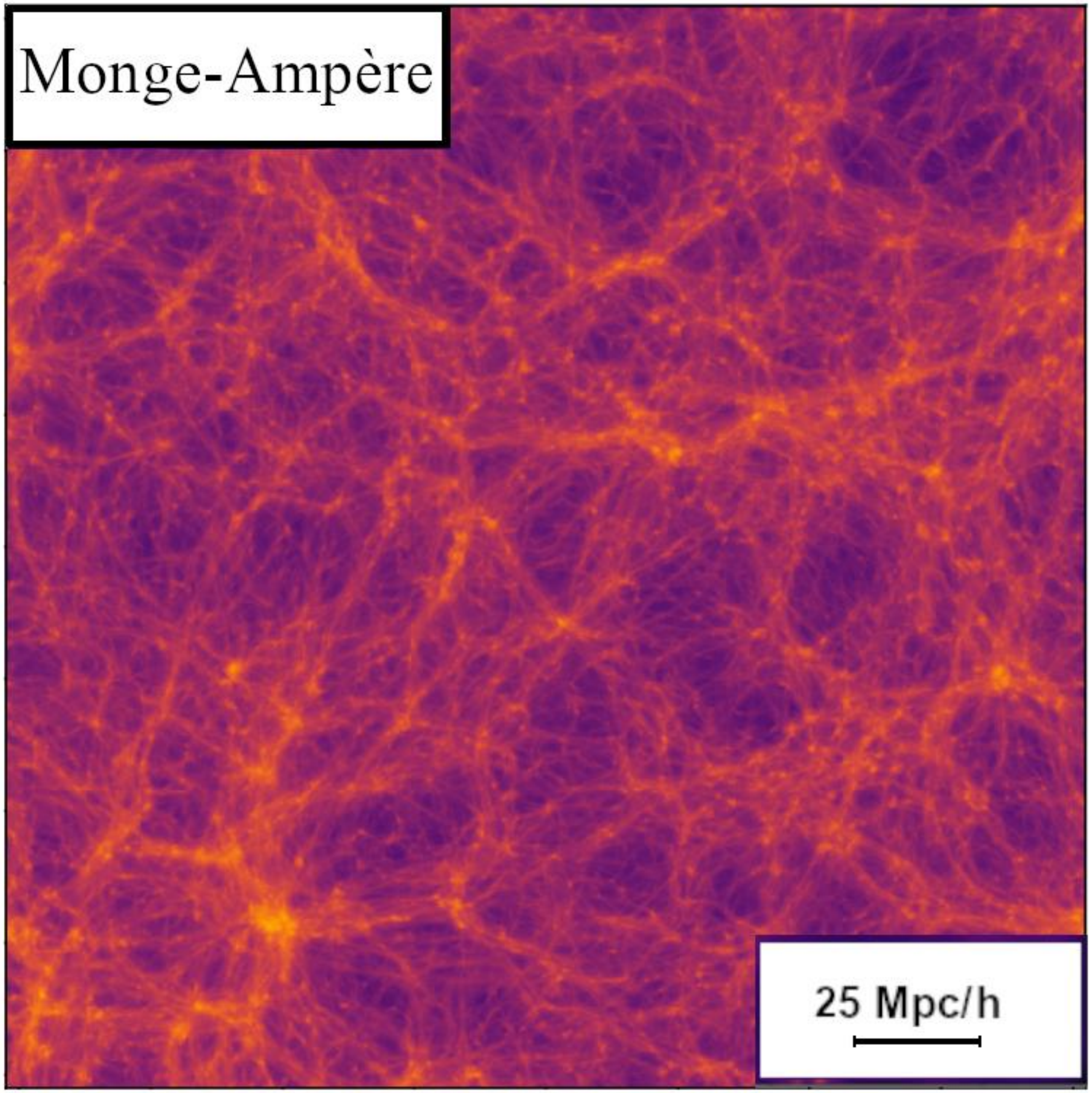

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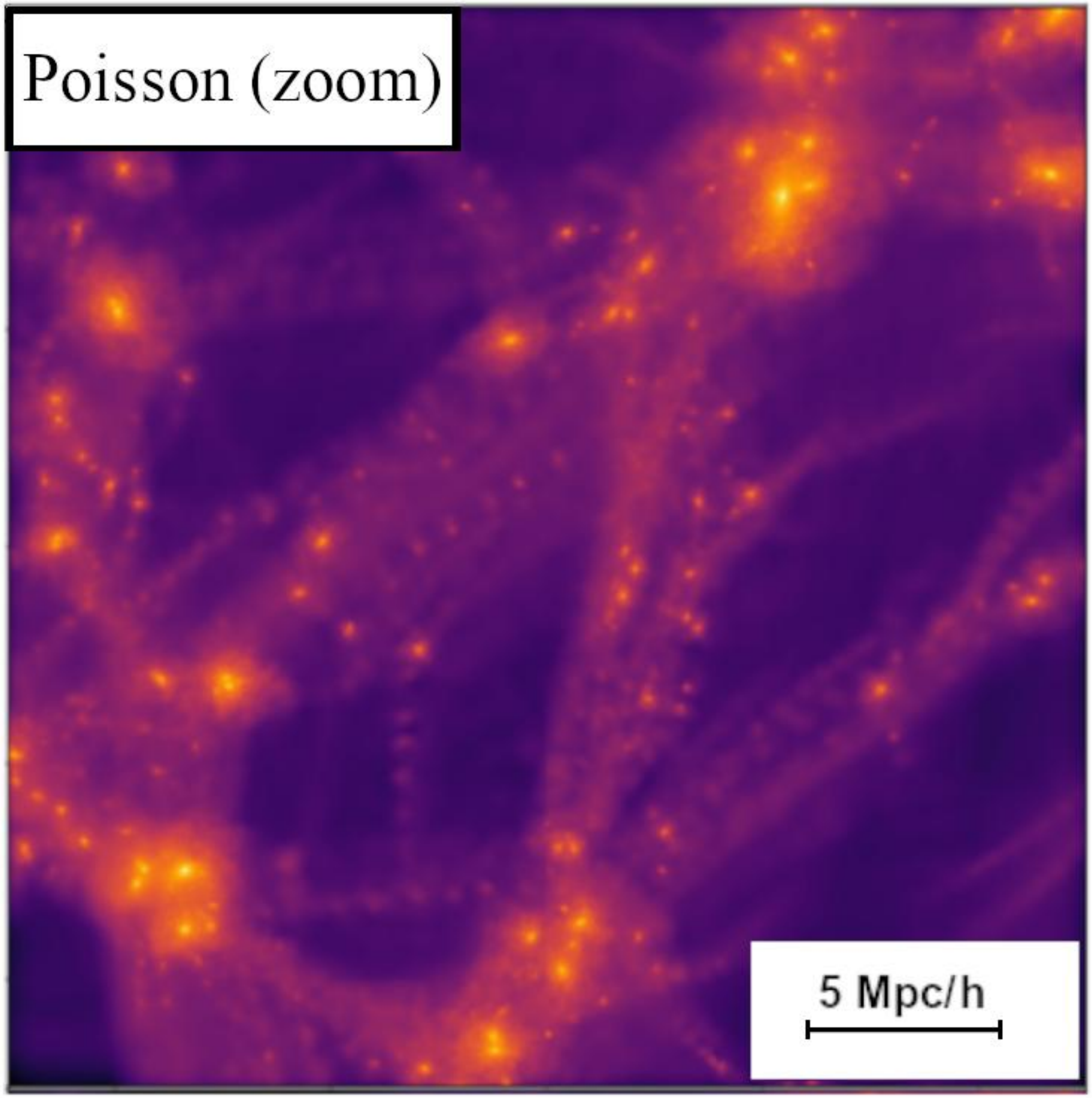

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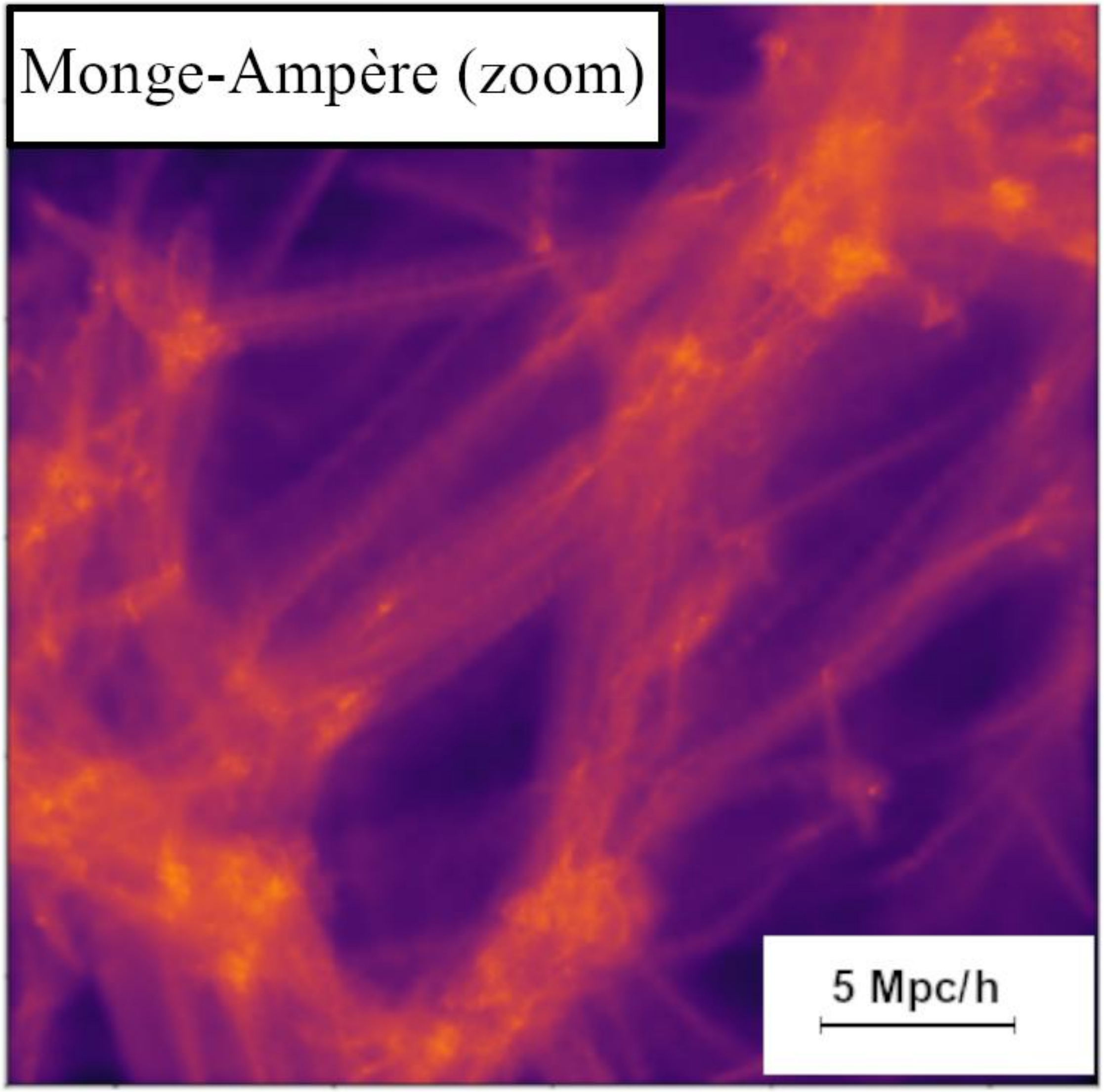

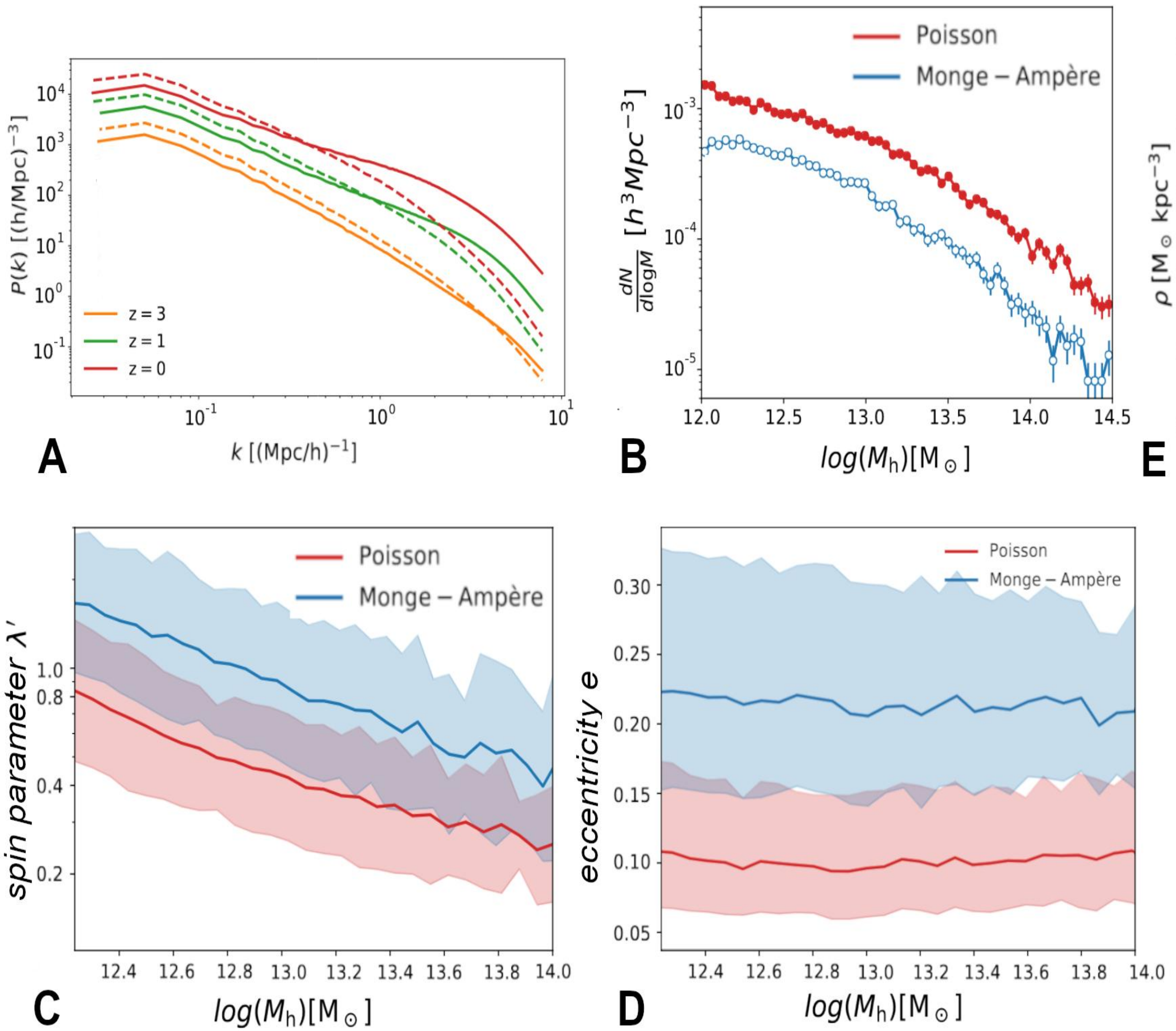

Halo masses
Halo shapes
Angular momentum
Rotation curves

Results – Conclusions

BMAG is a small *non-linear* modification of Newtonian dynamics

Differences:

- Larger number of filaments
- Smaller number of small haloes
- Haloes spin faster. Origin of angular momentum of disk galaxies ?
- Centrail density profile of haloes is flatter
- More power on large scales and less power on small scales

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Questions:

- BMAG as the weak field limit of another strong-field theory ?
- BMAG emerging from GR (or other modified theories of gravity) ?
- Entropic gravity ?

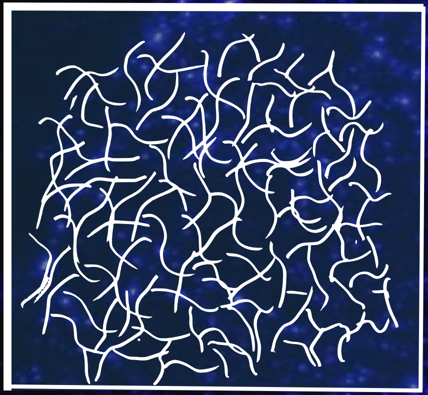
Future works:

Exploring the shape of the Universe

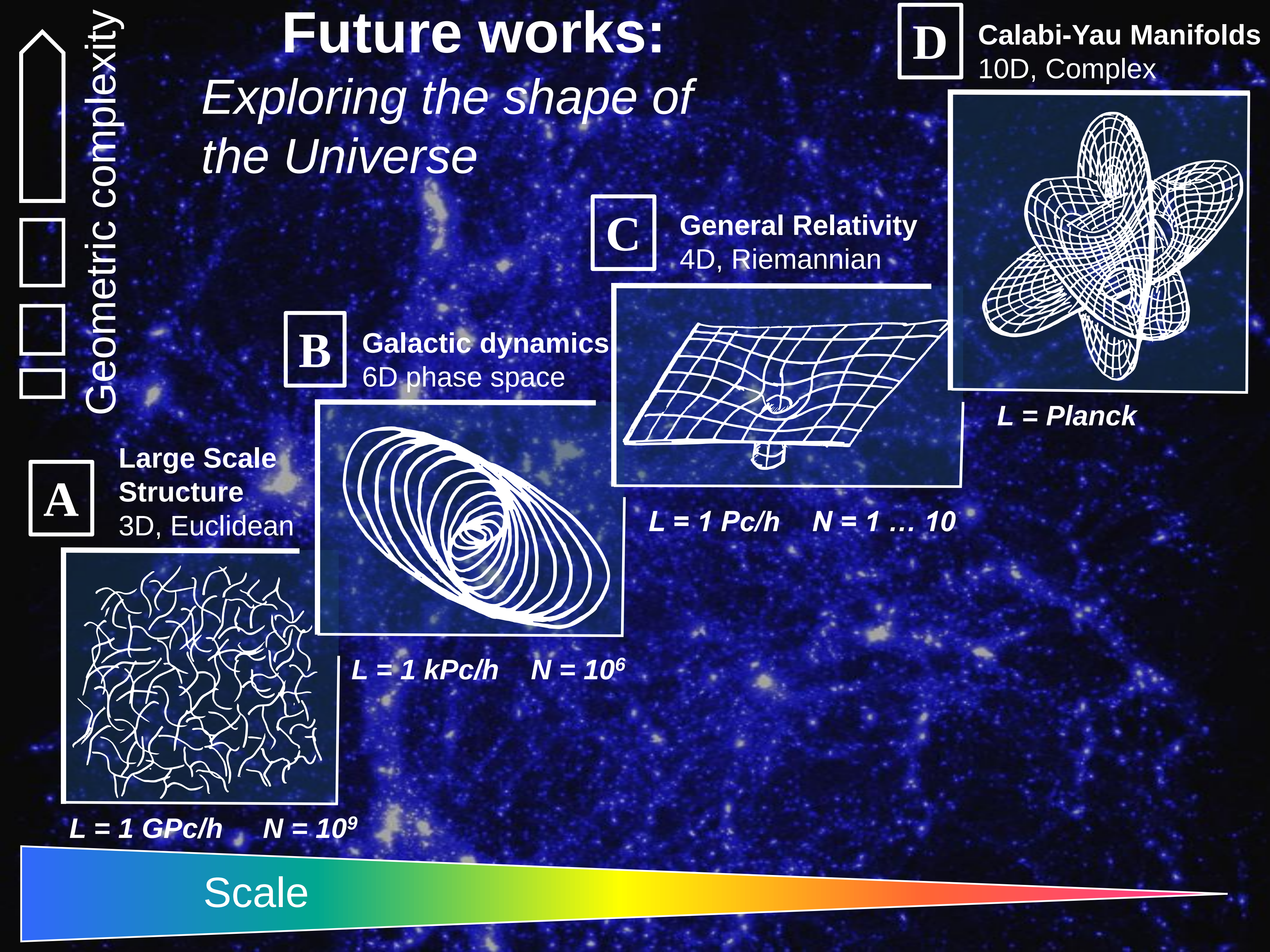
A

Large Scale Structure

3D, Euclidean



L = 1 GPc/h ***N = 10^9***


Future works:
Exploring the shape of the Universe
Geometric complexity
A
Large Scale Structure
3D, Euclidean
L = 1 GPc/h N = 10^9
B
Galactic dynamics
6D phase space
L = 1 kPc/h N = 10^6
C
General Relativity
4D, Riemannian
L = 1 Pc/h N = 1 … 10
D
Calabi-Yau Manifolds
10D, Complex
L = Planck
Scale

References on Cosmology and OT

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