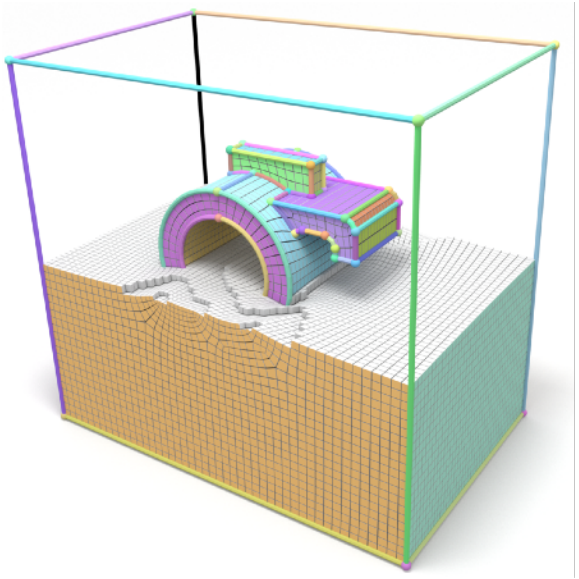
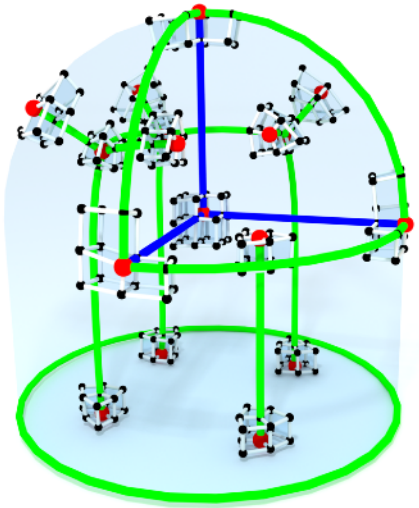
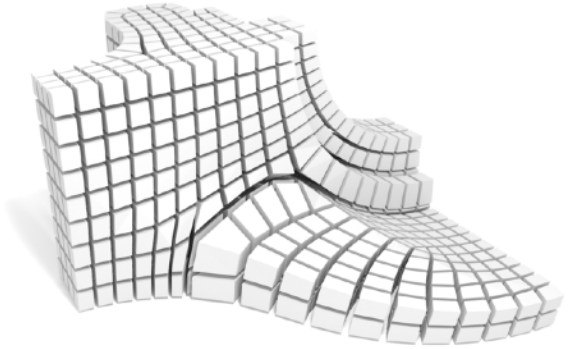
Quadrilateral and **Hexahedral** Mesh Generation with **Integer-Grid Maps**



David Bommes
University of Bern





European Research Council
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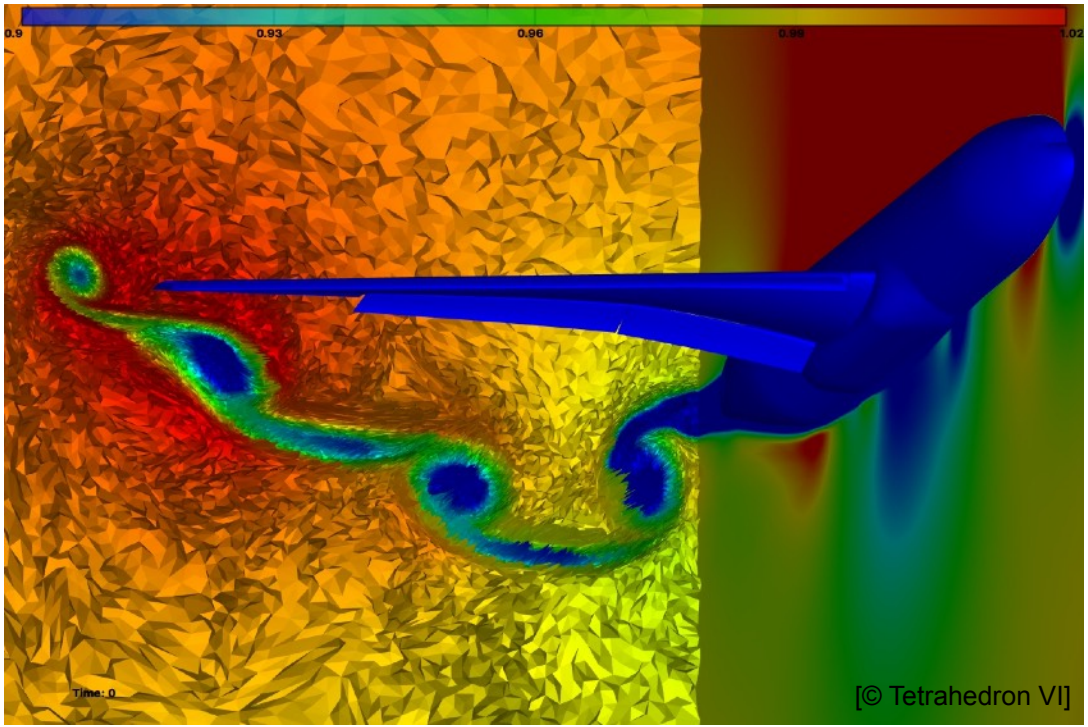
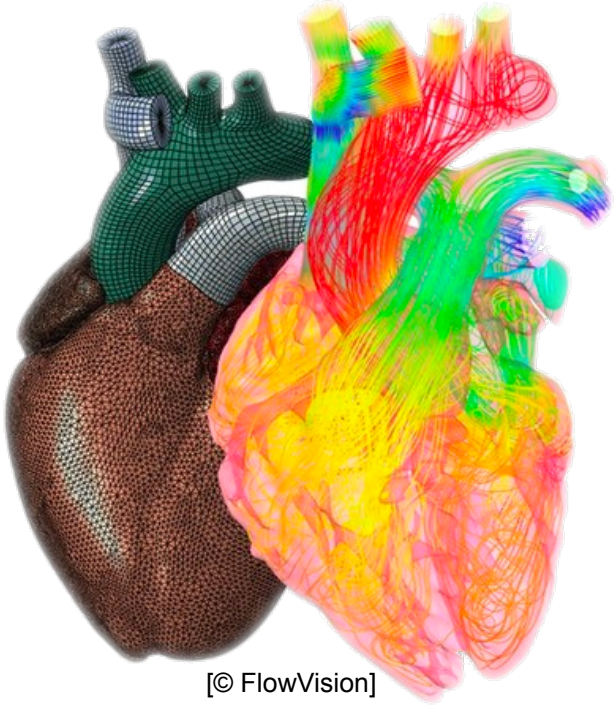
Context

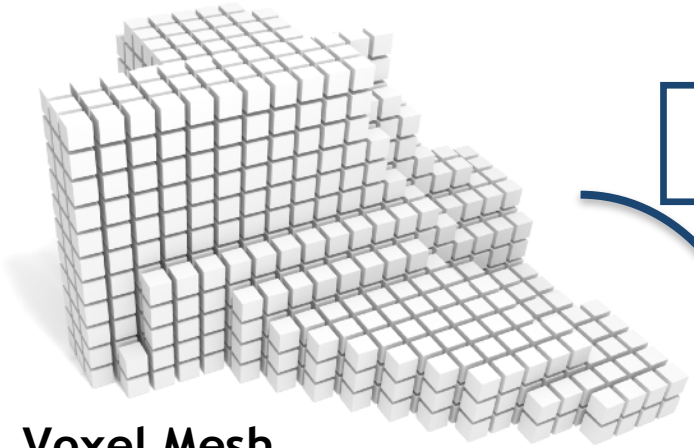
Simulation depends on **Volumetric Discretiztion**



[© Tetrahedron VI]


[© FlowVision]

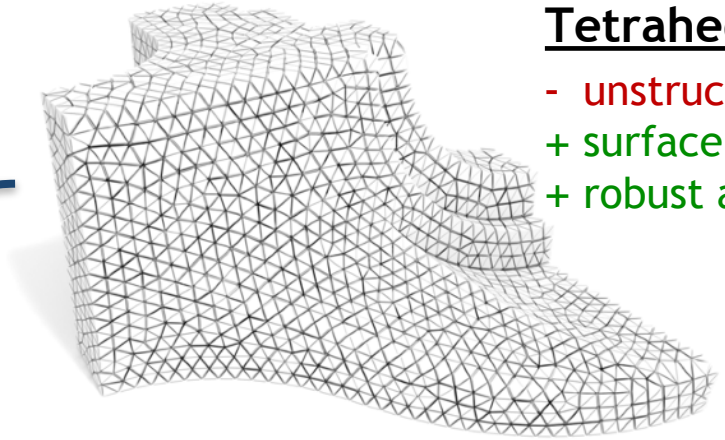
How to discretize volumetric domains?



Voxel Mesh

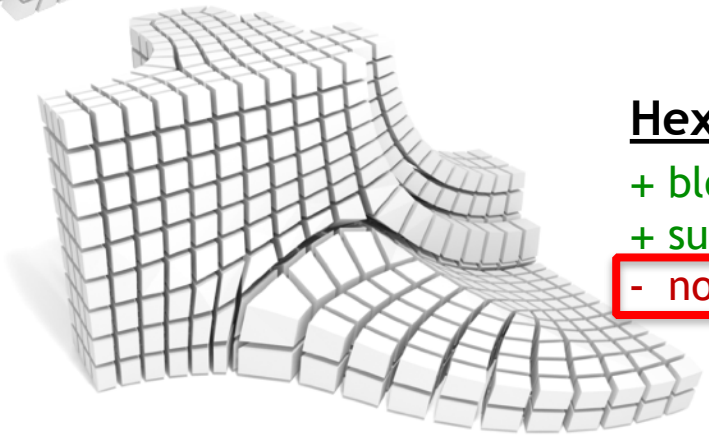
+ structured
- no surface alignment
+ trivial generation

combine strengths



Tetrahedral Mesh

- unstructured
+ surface alignment
+ robust algorithms



Hexahedral Mesh

+ block-structured
+ surface alignment
- no automatic algorithms

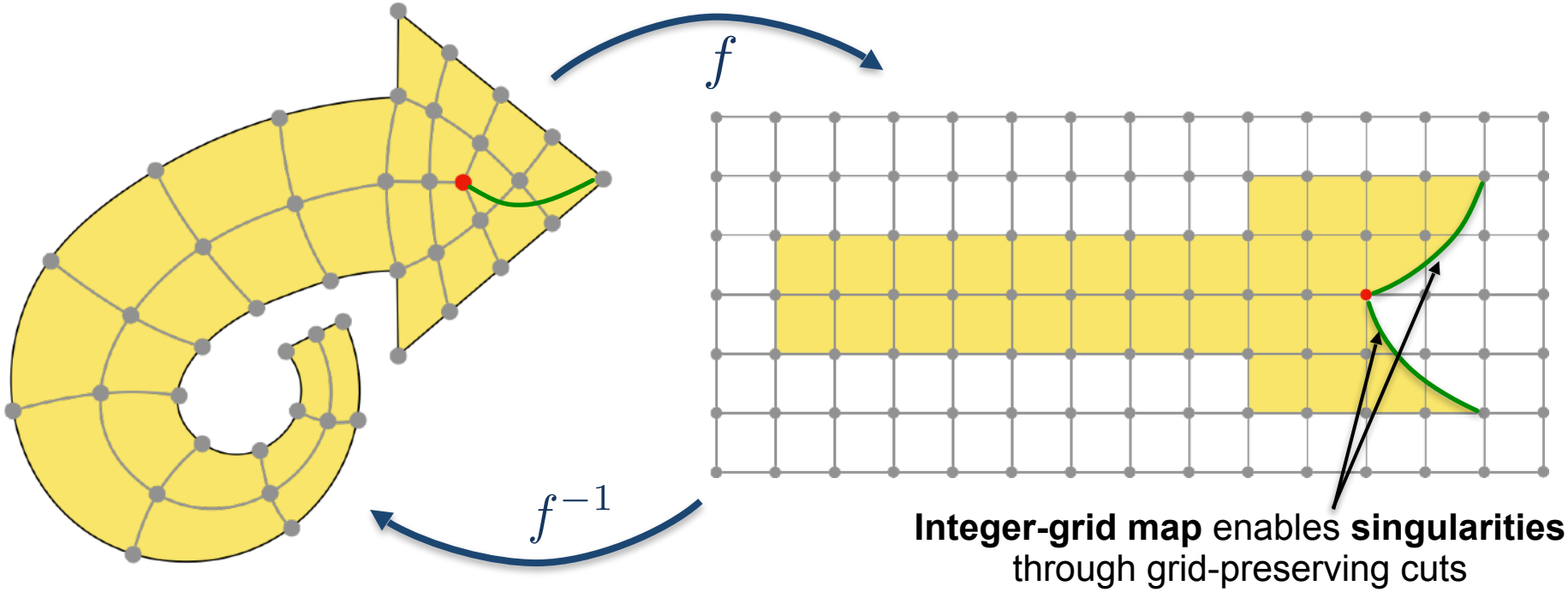
Research goal!

Integer-Grid Maps
Approach

Integer-Grid Maps

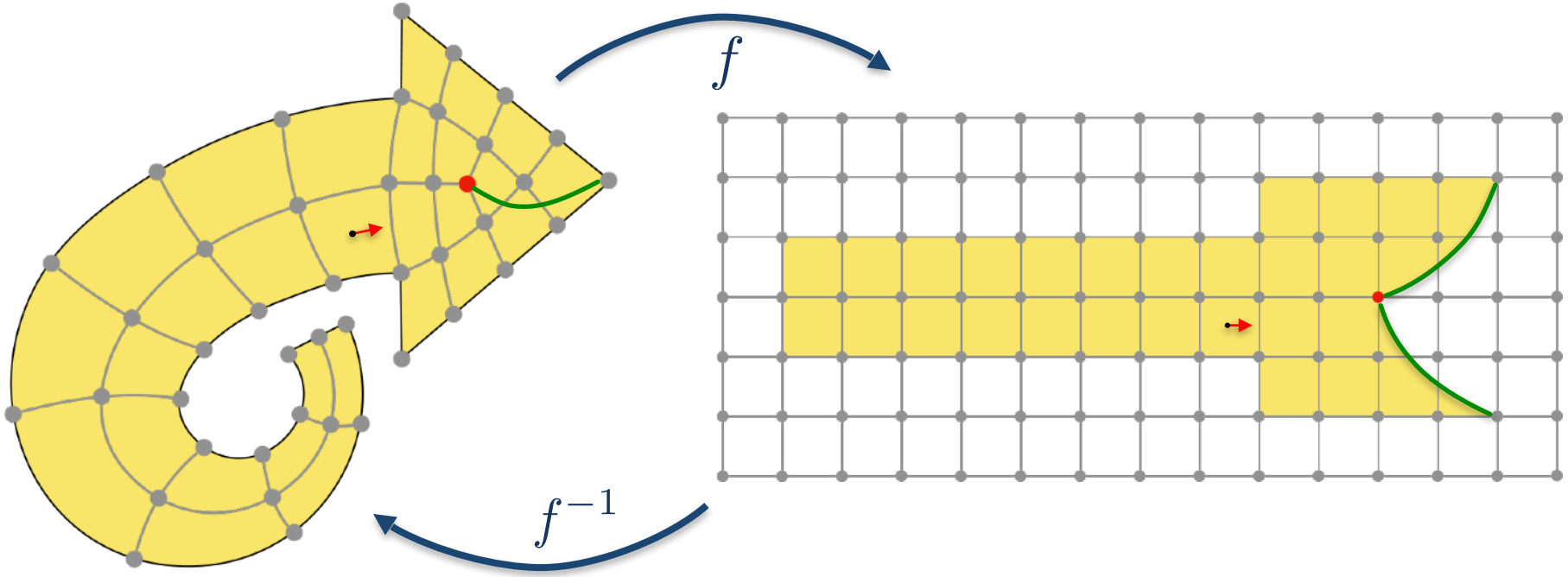
Idea: Interpret **Mesh Generation** as **Map Optimization**

Problem: **Map Construction** corresponds to **hard Mixed-Integer Problem**

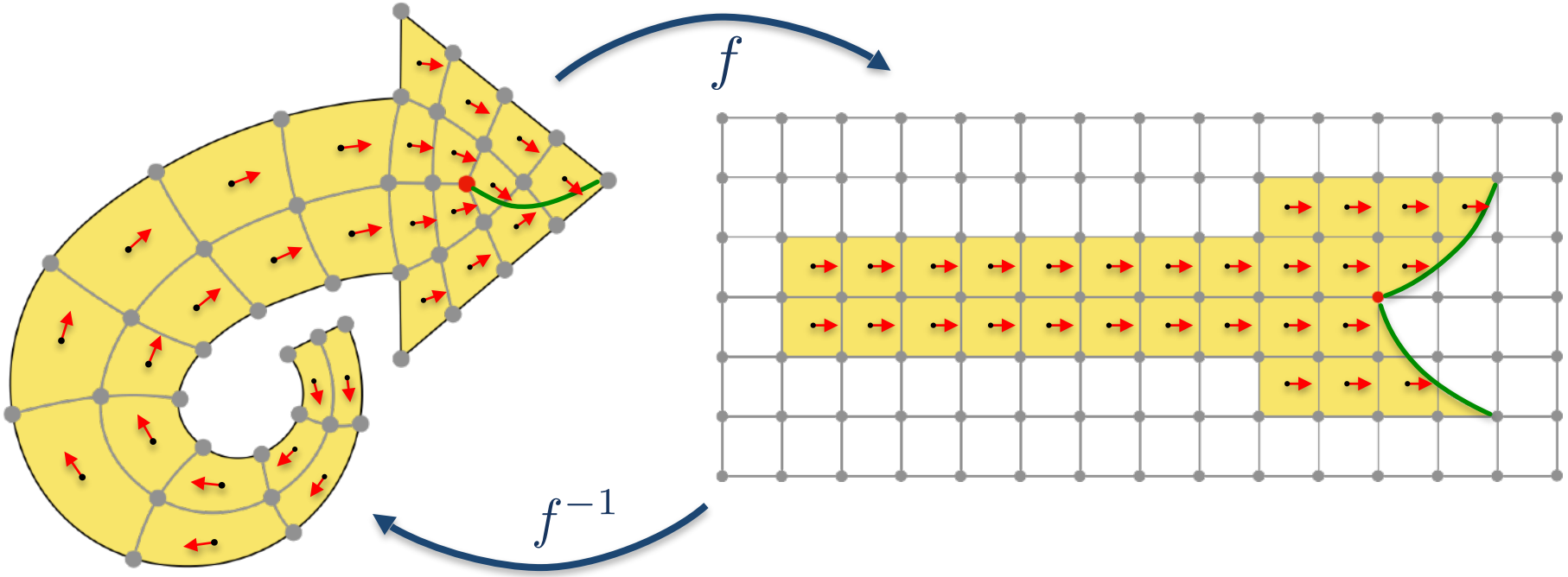


Integer-grid map enables **singularities** through grid-preserving cuts

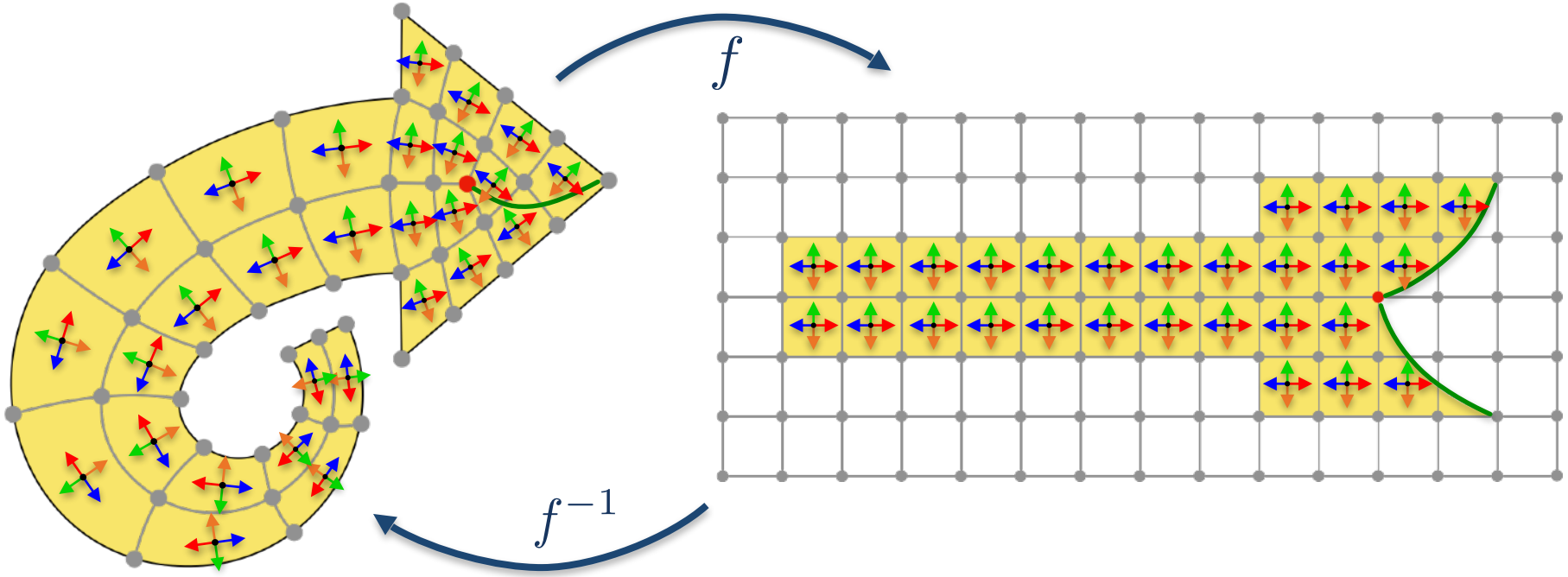
Frame Fields



Frame Fields



Frame Fields



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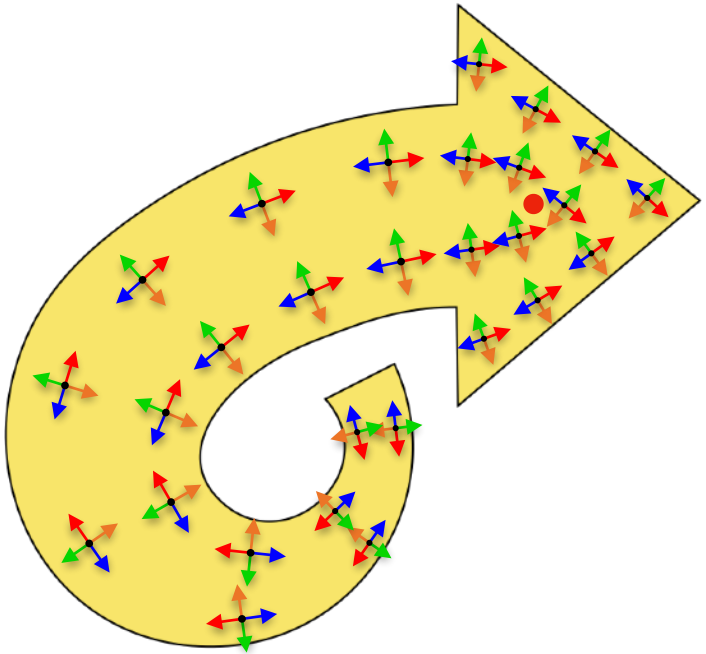


Idea:

Relax Integrability Constraint

➡ Search Smooth Frame Field

➡ Generate IG-Map with fixed Topology

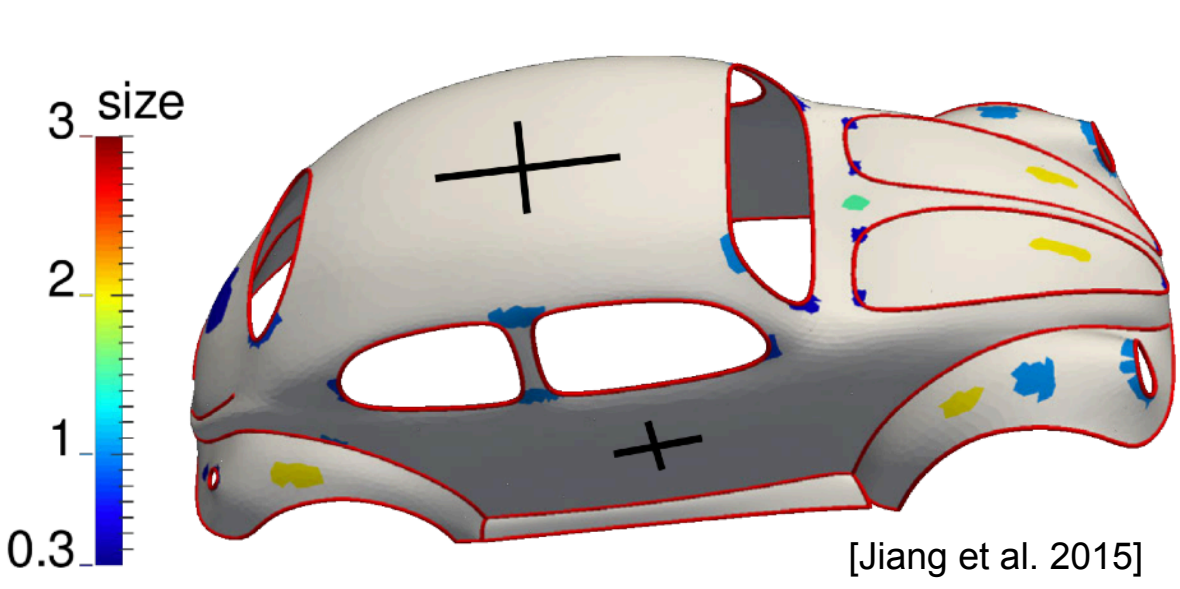
State of the Art
Quad Meshing with Integer-Grid Maps (IGMs)

joint work with Prof. Leif Kobbelt
Visual Computing Institute - RWTH Aachen

Practical Quad Meshing with IGMs

1. Specify guidance & constraints

- alignment to boundaries/features/curvatures
- element sizing & anisotropy
- singularities



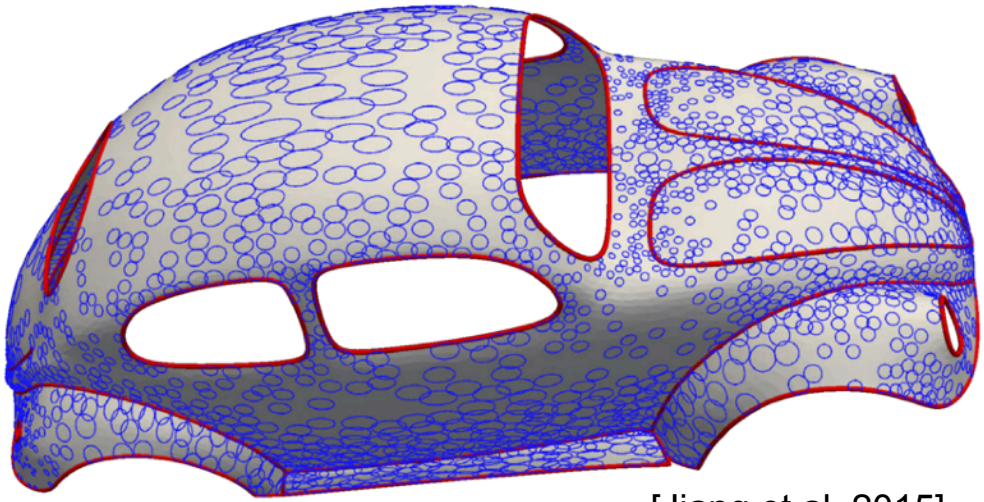
[Jiang et al. 2015]

Practical Quad Meshing with IGMs

1. Specify guidance & constraints

2. Generate metric field

- decomposition into orthonormal-field + metric simplifies optimization



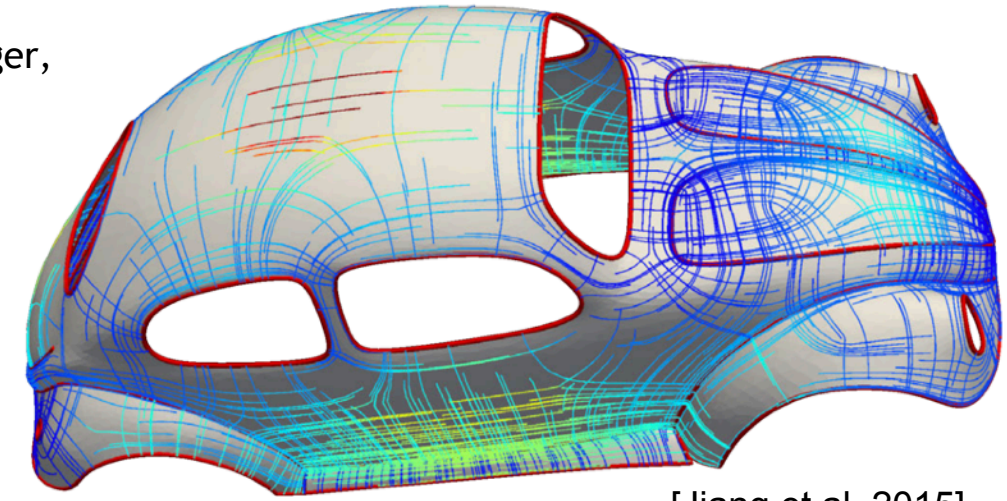
[Jiang et al. 2015]

Practical Quad Meshing with IGMs

1. Specify guidance & constraints

2. Generate metric field

3. Generate frame field
   - many different algorithms (mixed-integer, polyvector fields, MBO, …)
   - smoothness, fitting and integrability



[Jiang et al. 2015]

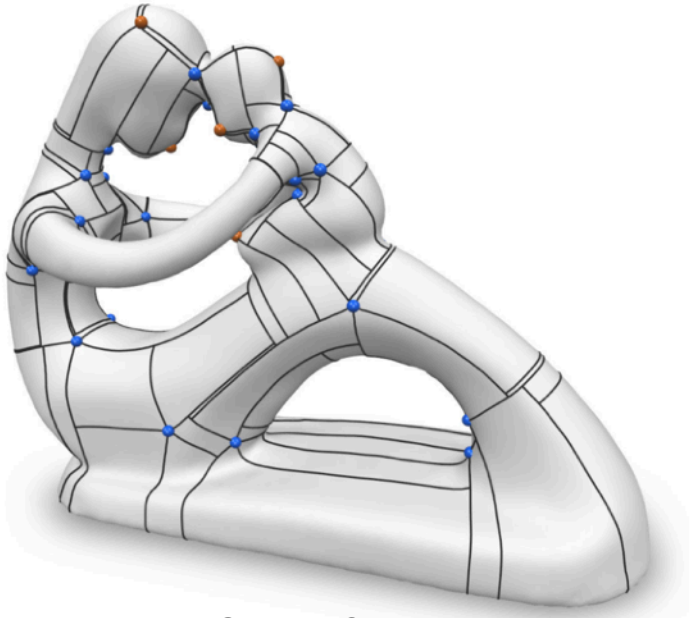
Practical Quad Meshing with IGMs

1. Specify guidance & constraints

2. Generate metric field

3. Generate frame field

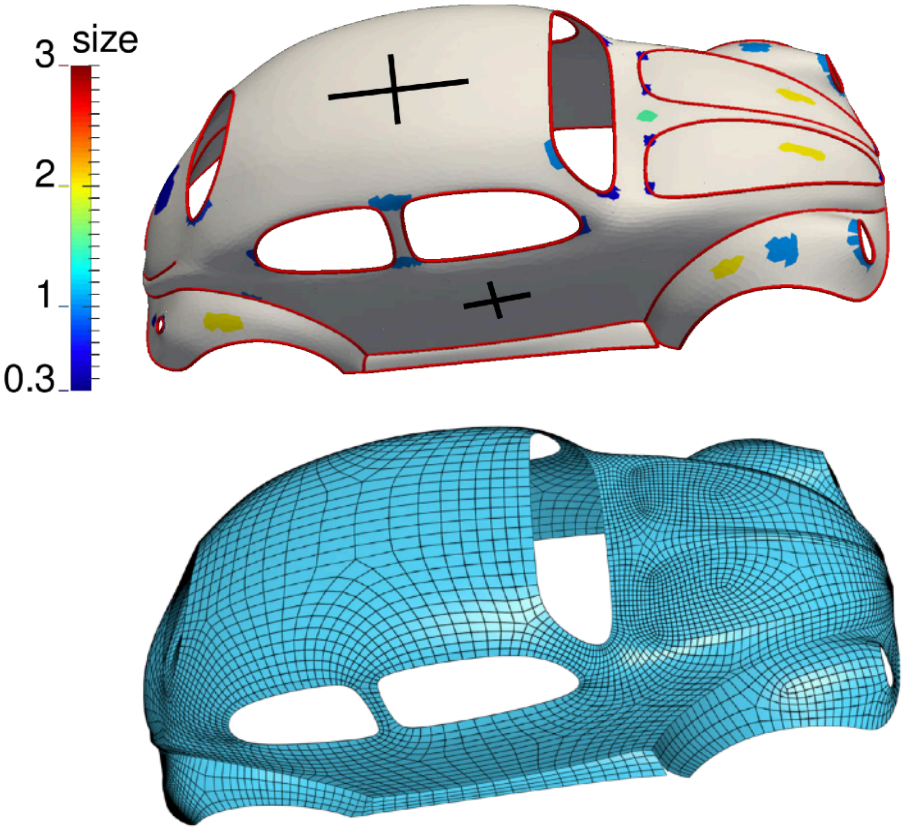
4. Generate Integer-Grid Map
   - trace motorcycle graph
   - quantize resulting T-Mesh
   - optimize locally injective IGM subject to quantization constraints



Motorcycle Graph [Campen et al. 2015]

Practical Quad Meshing with IGMs

1. Specify guidance & constraints
2. Generate metric field
3. Generate frame field
4. Generate Integer-Grid Map
5. Mesh extraction / higher-order fitting



[Jiang et al. 2015]

Success Story of Integer-Grid Maps in Quad Meshing

Technology Transfer based on

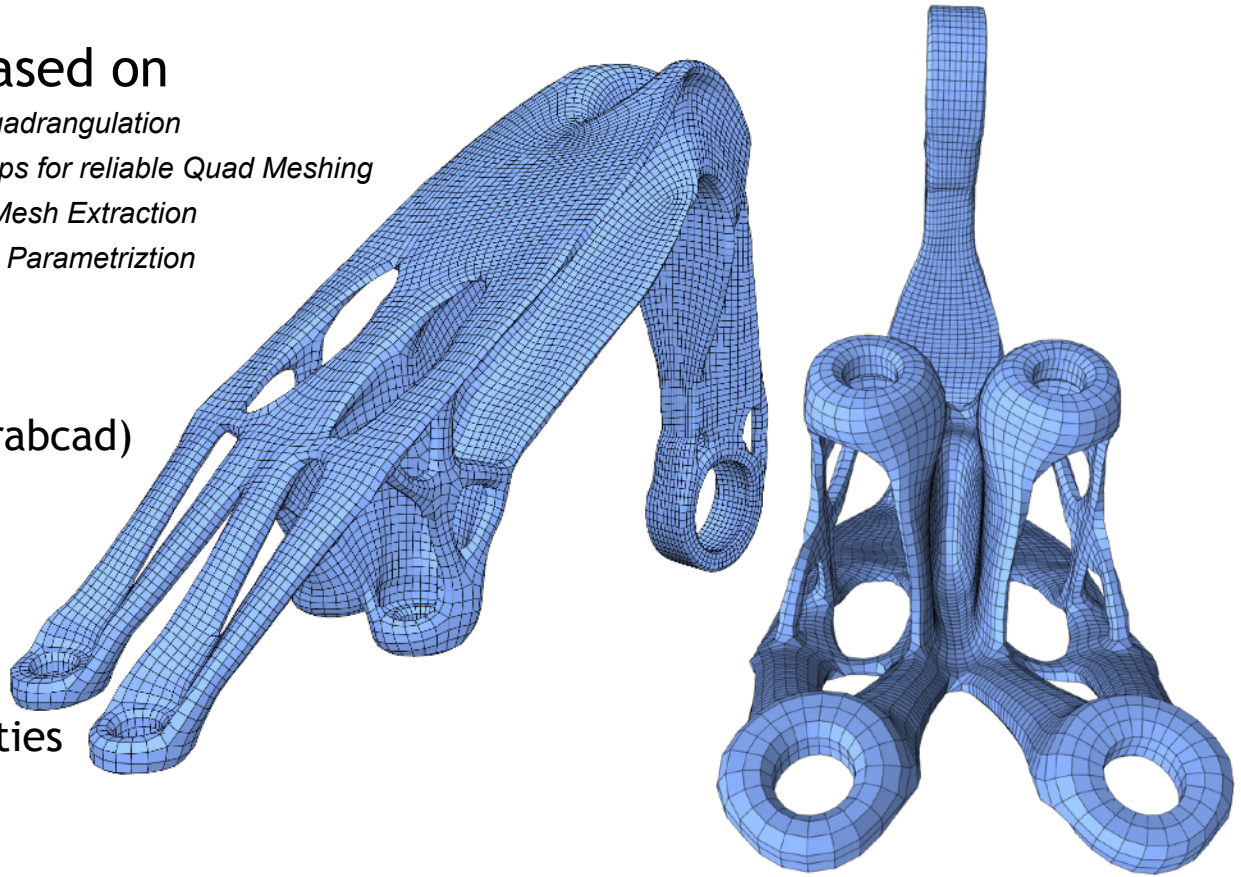
- [Bommes et al. 2009] – *Mixed-Integer Quadrangulation*
- [Bommes et al. 2013a] – *Integer-Grid Maps for reliable Quad Meshing*
- [Ebke et al. 2013] – *QEX: Robust Quad Mesh Extraction*
- [Campen et al. 2015] – *Quantized Global Parametriztion*

Example Model:

- airplane bearing bracket (grabcad)
- #triangles = 215k
- genus = 19

Output:

- one-click solution
- globally optimized singularities
- runtime **80s** (#quads = 17k)



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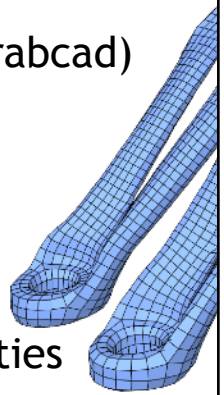
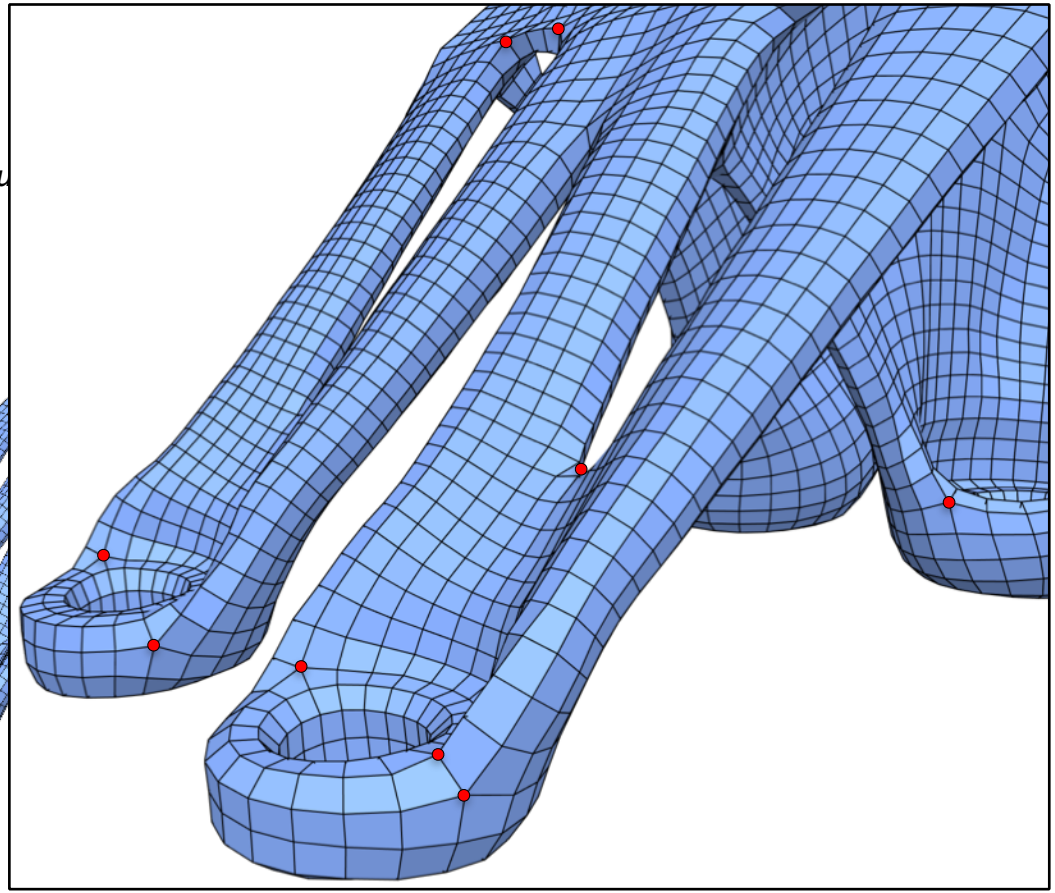


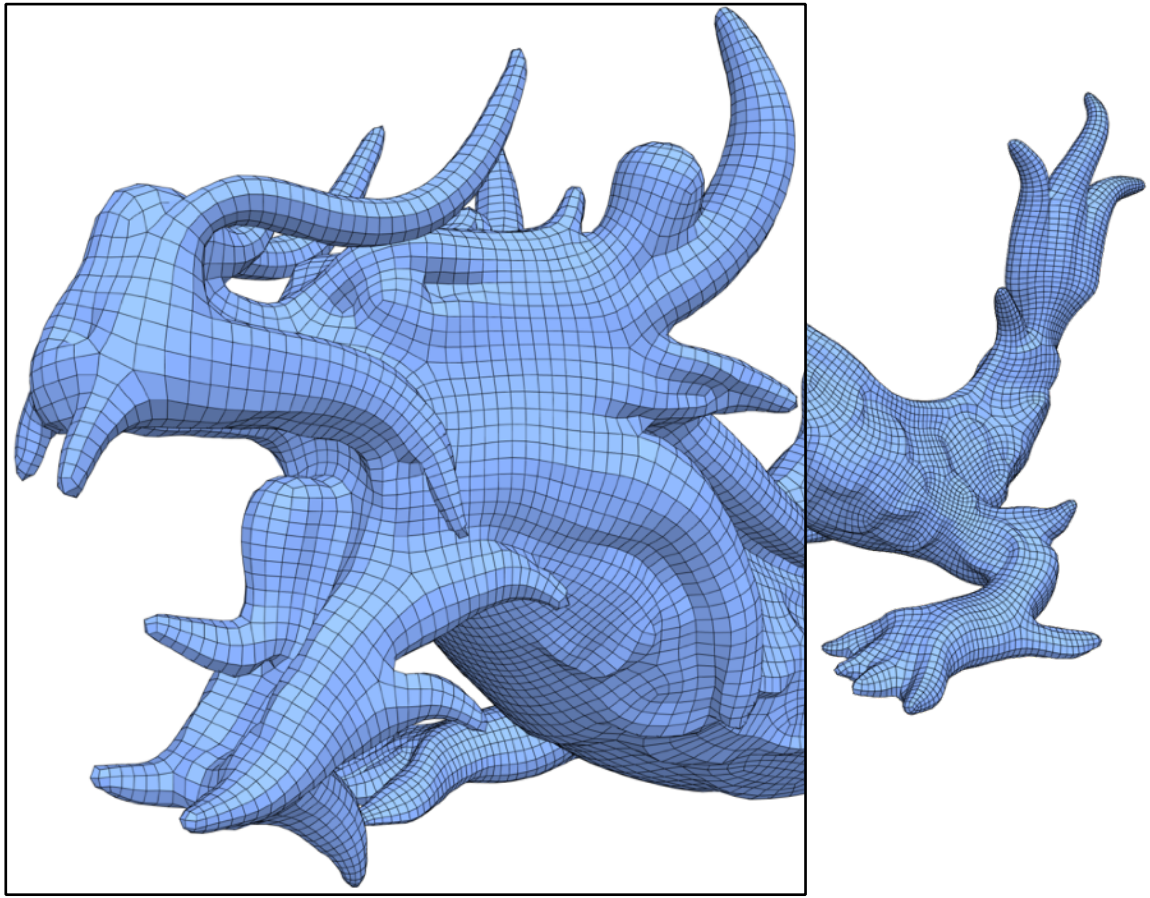


Results — one-click solution

- asian dragon

- **input**:
  - #triangles 140k
  - many geometric details

- **output**:
  - #singularities 624
  - #quads 27k
  - runtime **90s**

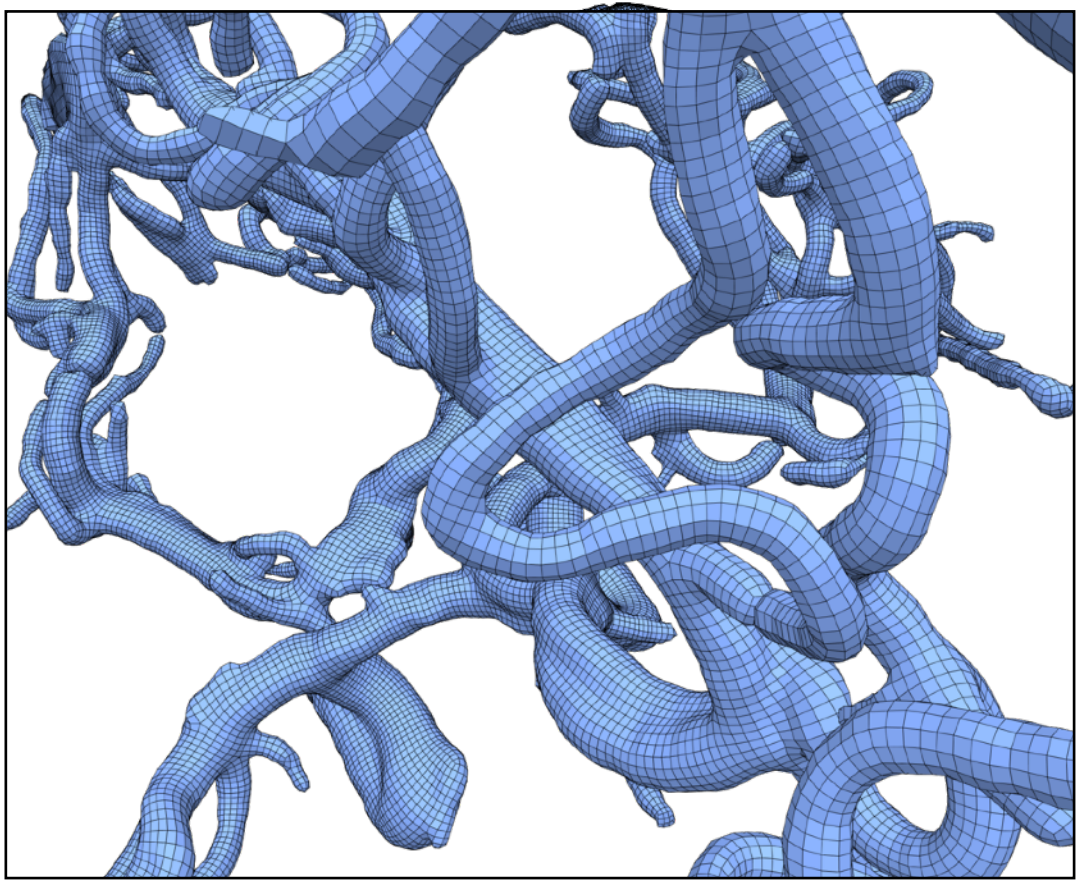


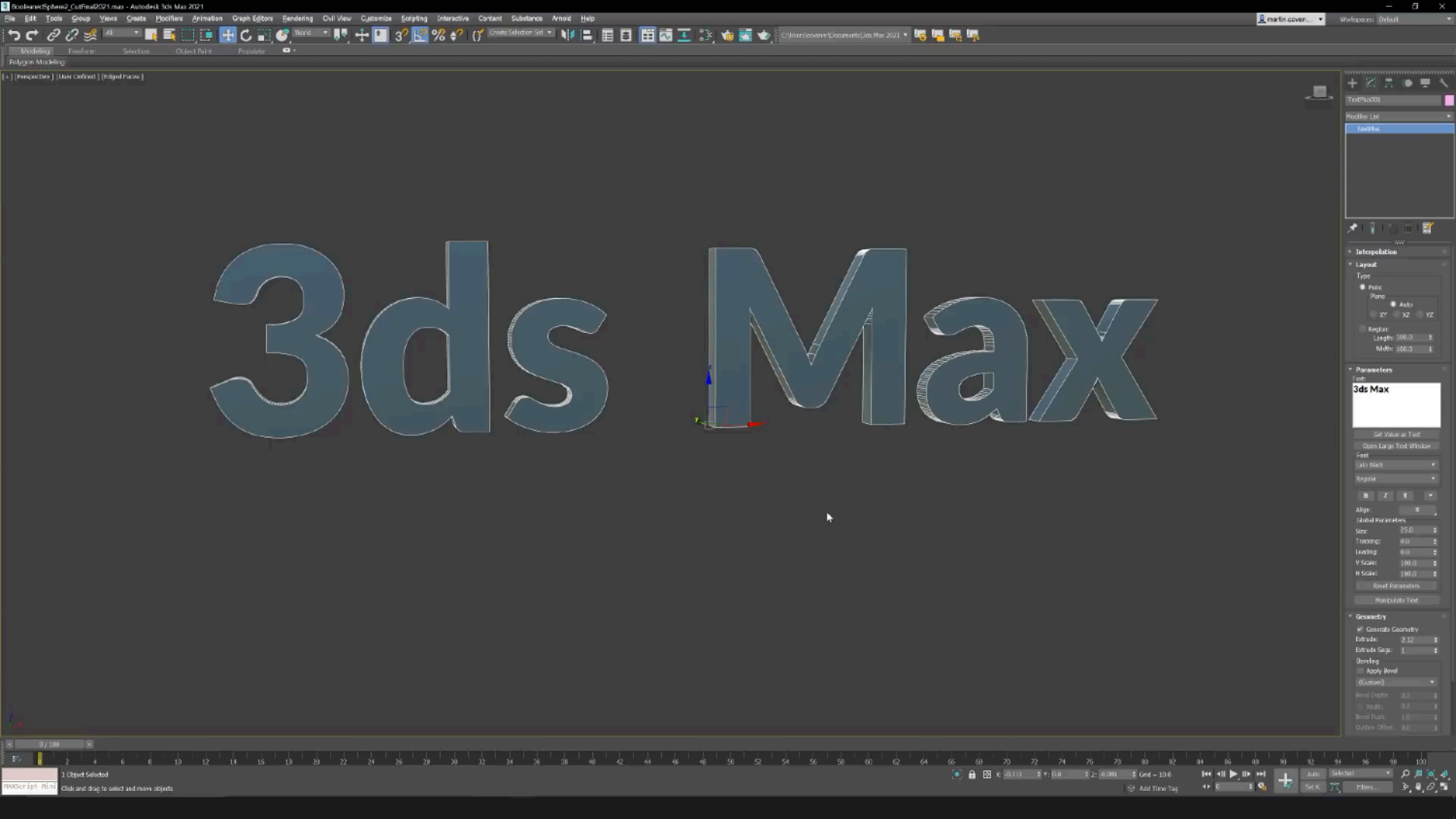
Results — one-click solution

- vascular structure

- **input**:
  - #triangles 224k
  - tubular network

- **output**:
  - #singularities 1851
  - #quads 125k
  - runtime **3.8min**




3ds Max

Autodesk 3ds Max Demo

- **Youtube:** https://www.youtube.com/watch?v=ktrIhk9NFT8
- **Author:**  martin coven 2.44K subscribers (Principal Experience Designer at Autodesk)
- **Selected Comments:**

Luke Costello 4 months ago

please put this in blender. This is absolutely amazing. This looks like neural networks have got to be involved.

Jixal 4 months ago (edited)

Really awesome stuff! Looks like it doesn't do the horrible spiral loops that other auto retopo tools tend to do =).

Chronis Art 4 months ago

Brilliant! Congratulations to the team, I have been trying it today.... Great to see 3dsMax 2021 is such a worthwhile upgrade! THANK YOU

Stuffy stuff 4 months ago

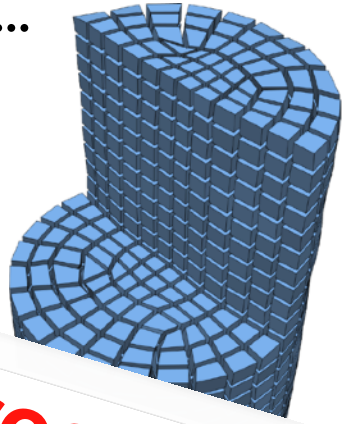
The closest autoretopo tool out there that matches in quality this new approach is quadremesher, sadly it doesn't work as well with details of multiple sizes, that crab would be a nightmare-is scenario, if you are down to port this new tool to blender I would gladly pay for it :) !

What about **Hexahedral Meshing** with IGMs?

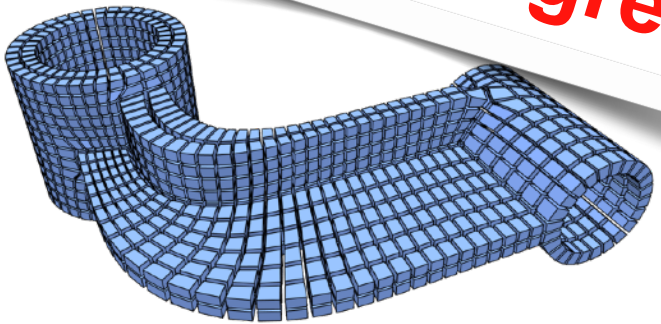
State of the Art – Hexahedral Meshing with IGMs

What we can mesh…

- single object
- single scale
- simple geometry

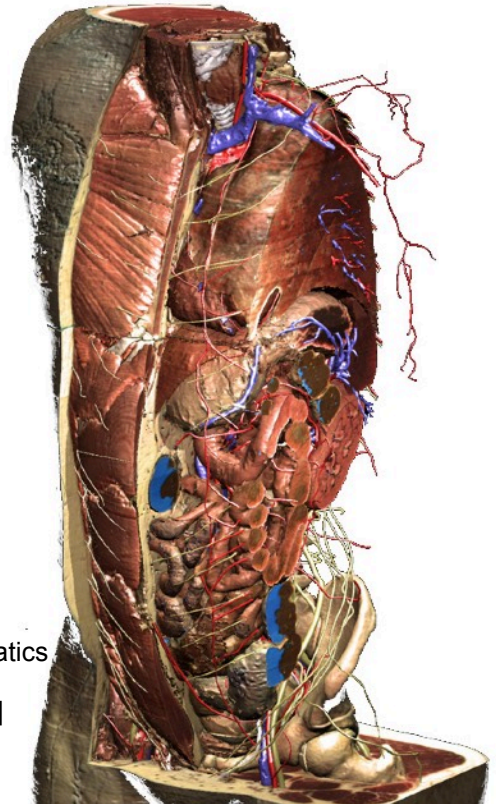


Recent Progress!



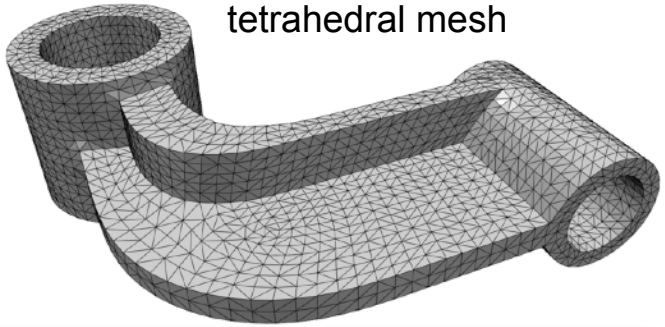
What we would like to mesh …

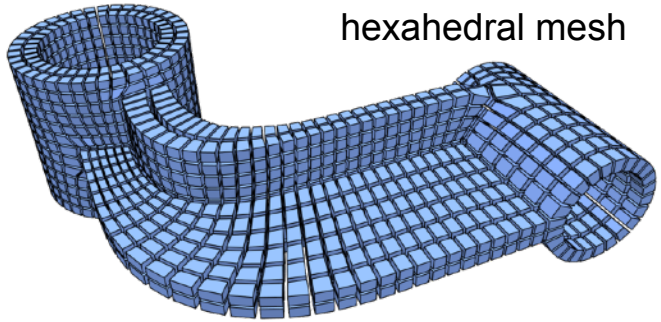
- nested objects
- multiple scales
- complex geometry



[© Institute of Mathematics and CS in Medicine - University of Hamburg]

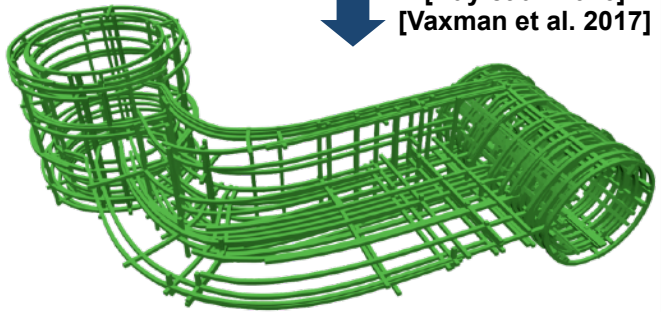
Hexahedral Meshing via IGM





problematic step

[Huang et al. 2011]
[Li et al. 2012]
[Jiang et al. 2014]
[Ray et al. 2016]
[Vaxman et al. 2017]

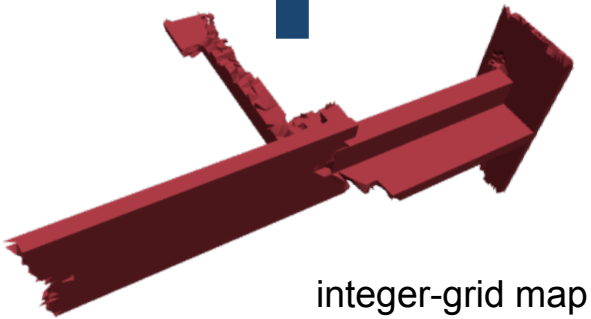


boundary aligned frame field

[Gao et al. 2017]
[Chemin et al. 2018]
[Palmer et al. 2019]
[Golovaty et al. 2019]
[Beaufort et al. 2019]

[Nieser et al. 2011]

[Lyon et al. 2016]

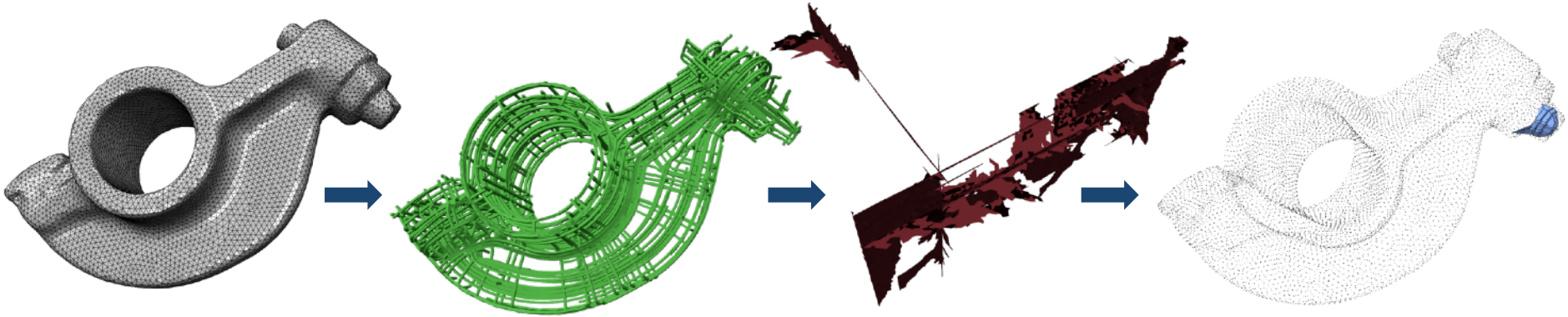


integer-grid map

Fundamental Topological Problem

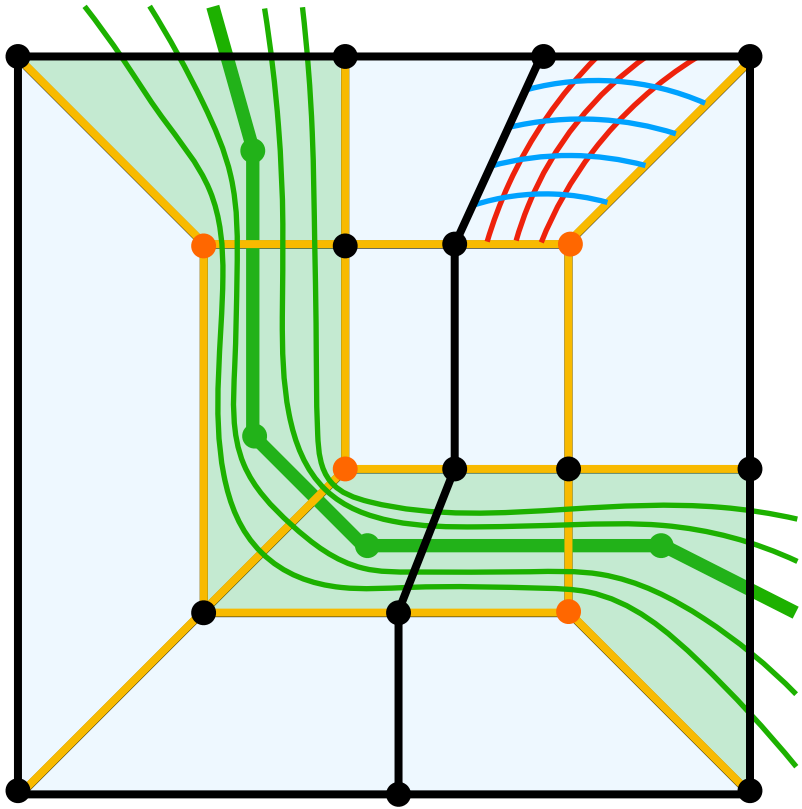
hexahedral mesh singularities ⊂ **frame field singularities**

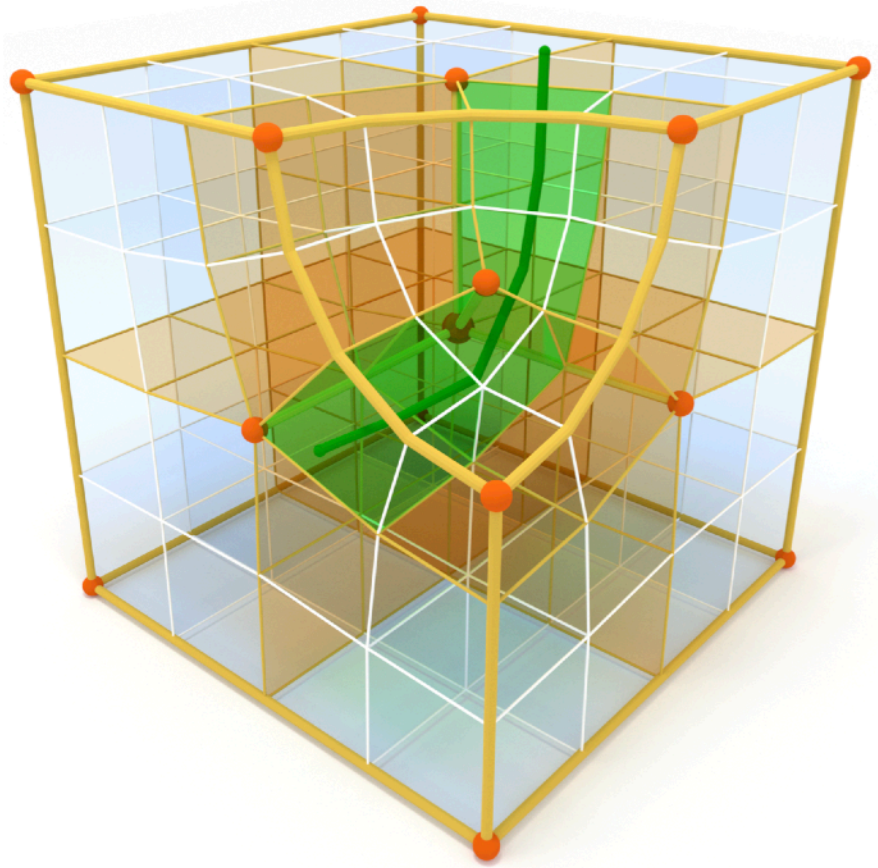
invalid singularities ➡ integer-grid map degenerates



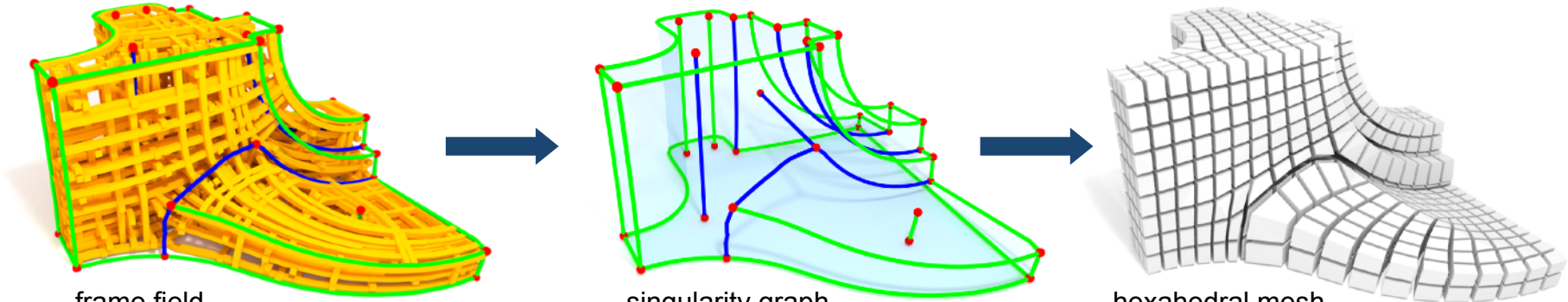
Frame Field
Meshability

Mesh-Induced Frame Field





Global Meshability



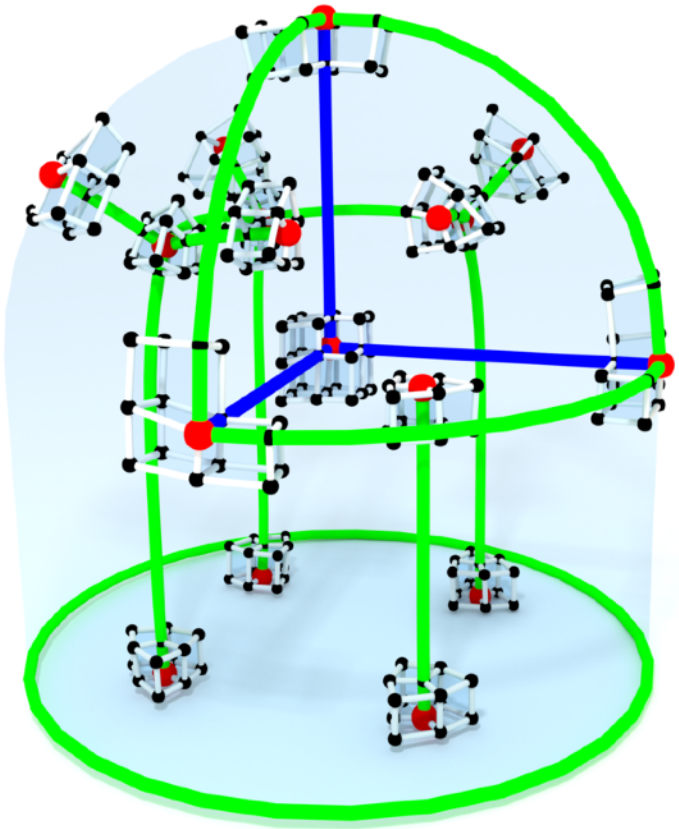
Frame Field
(globally) meshable

Hexahedral mesh of identical topology exists

Local Meshability

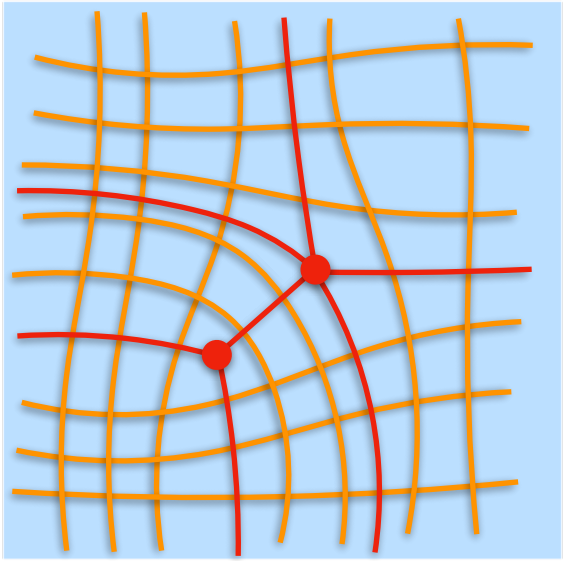
Frame Field
locally meshable

At each point of the domain there is a meshable neighborhood



Frame Field Meshability 2D

Quad Mesh vs. Frame Field Topology



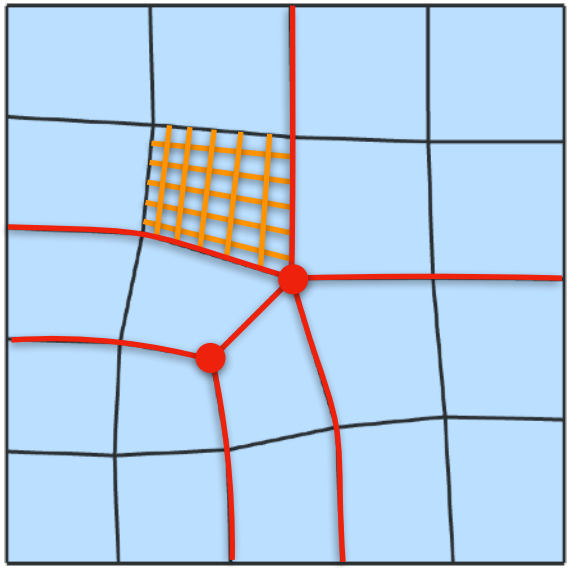
Frame Field

Induced Field

always

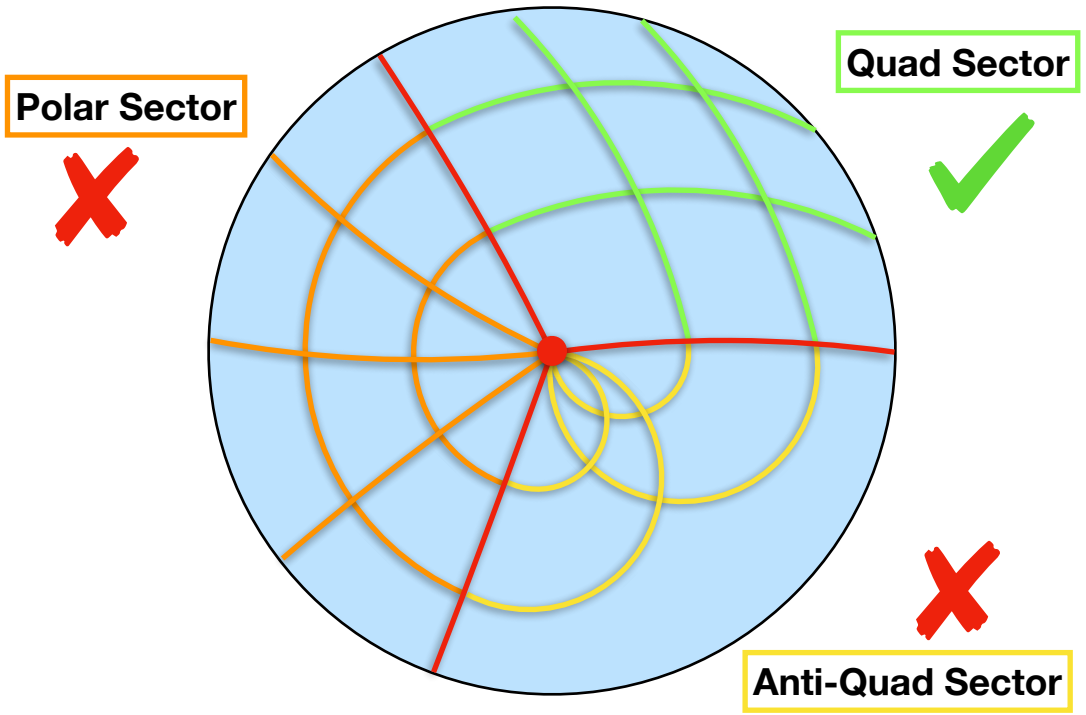
sometimes

Meshability

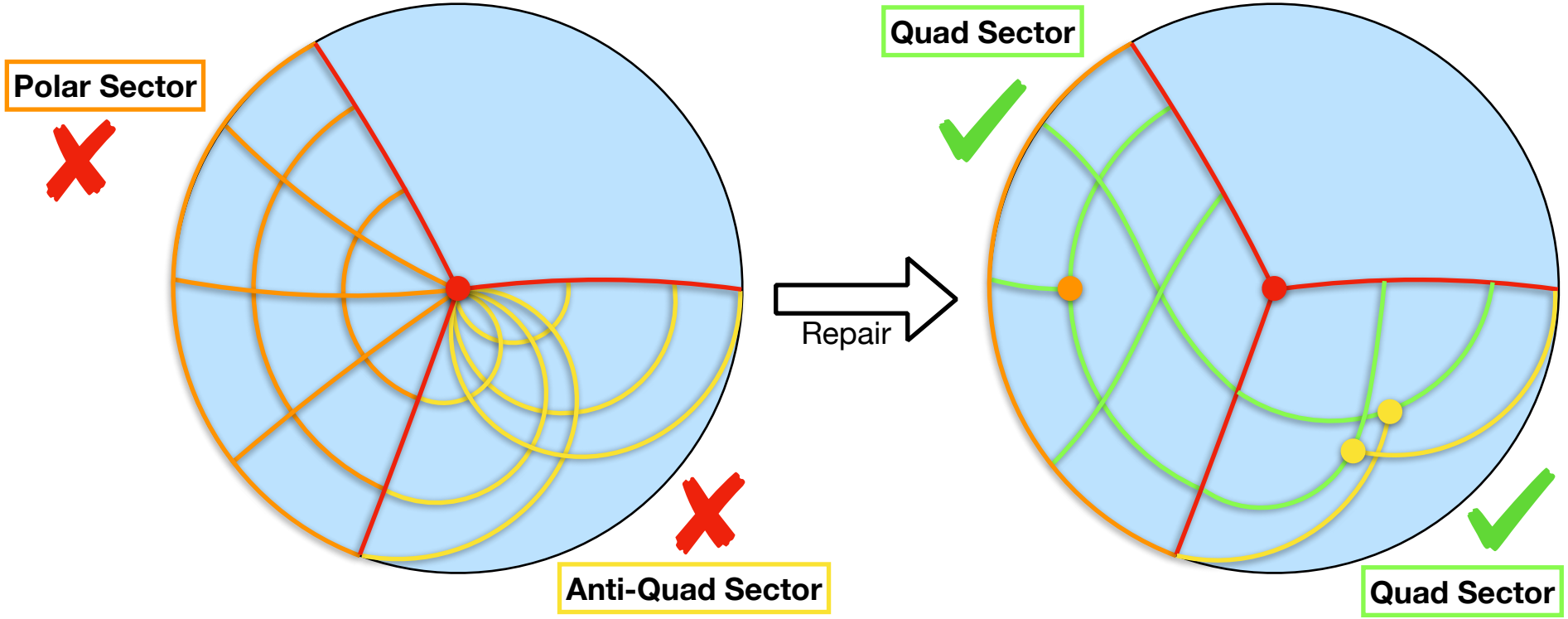


Quad Mesh

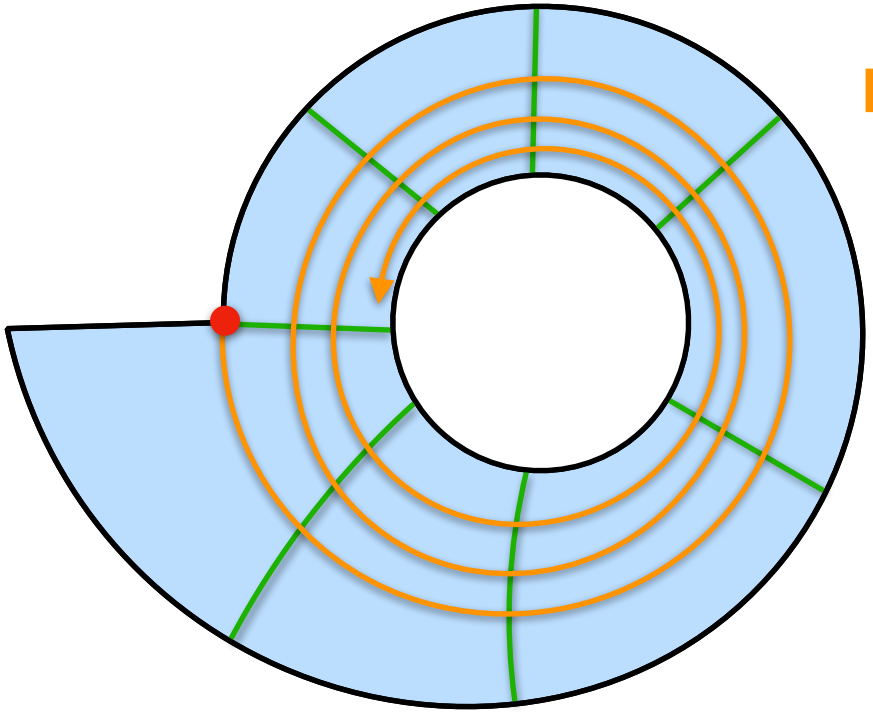
Local Meshability Condition



Local Meshability Repair

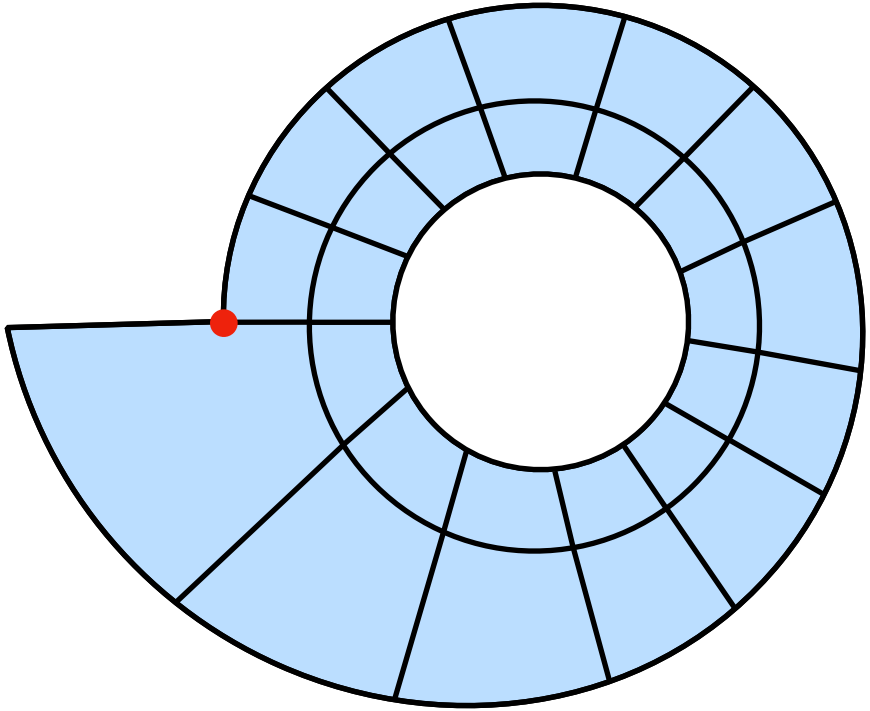


Local vs. Global Meshability



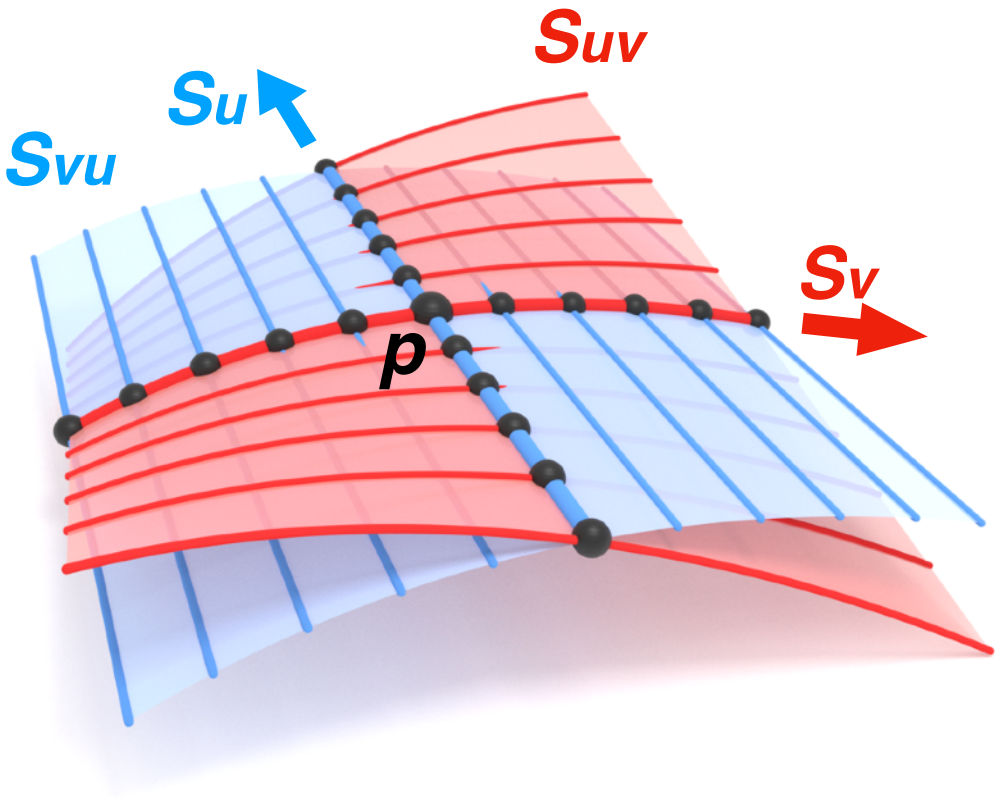
Limit Cycle

Local vs. Global Meshability



Frame Field Meshability 3D

Non-Integrability of 3D Frame Fields

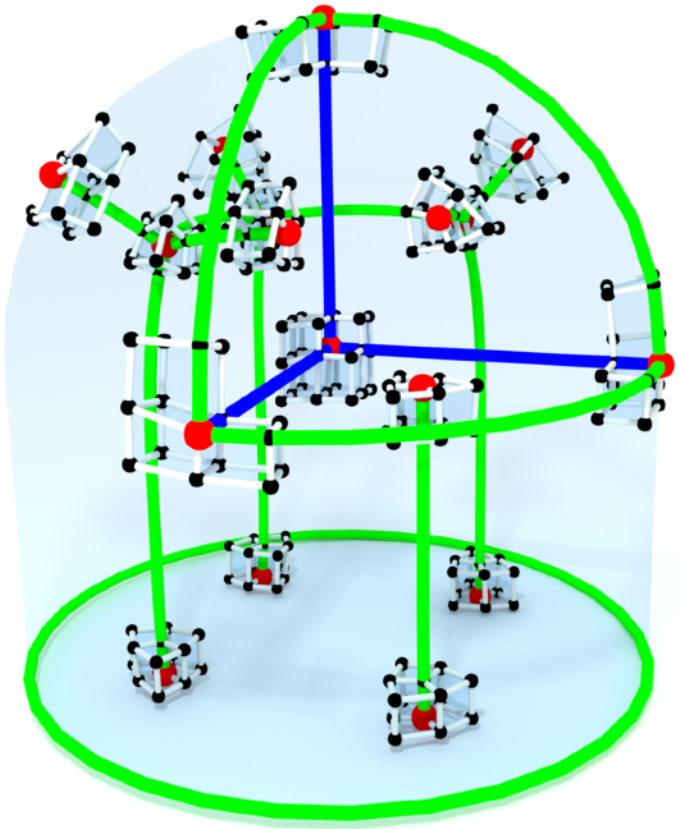


Local Meshability

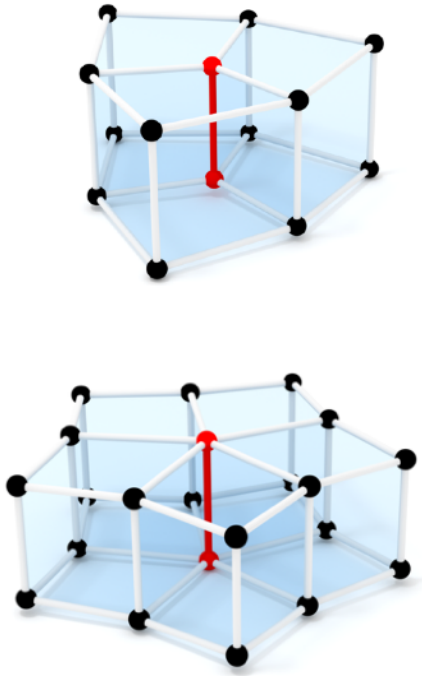
All sectors are hyperbolic

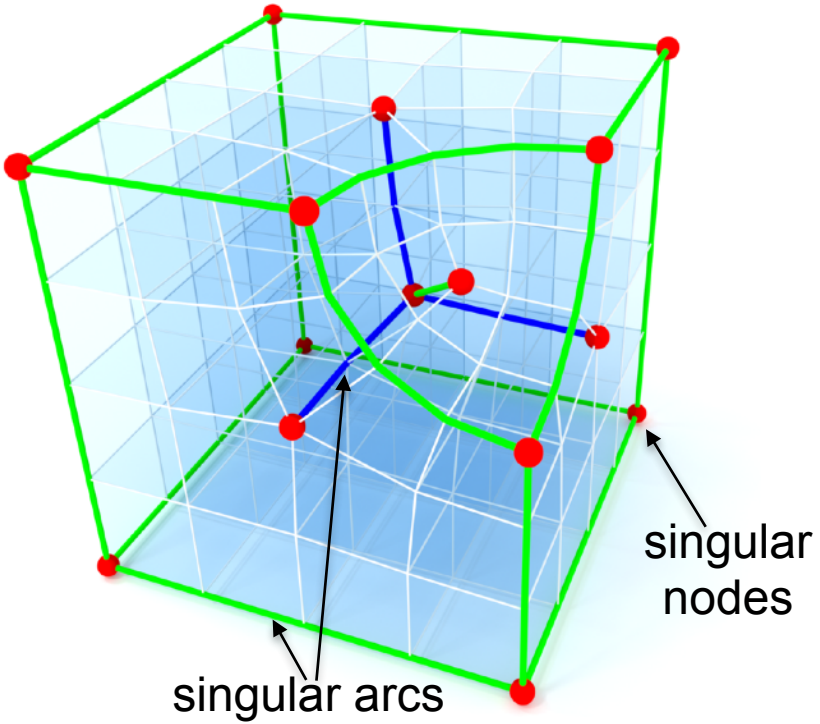
Frame Field
locally meshable

At each local neighborhood an IGM of identical topology exists

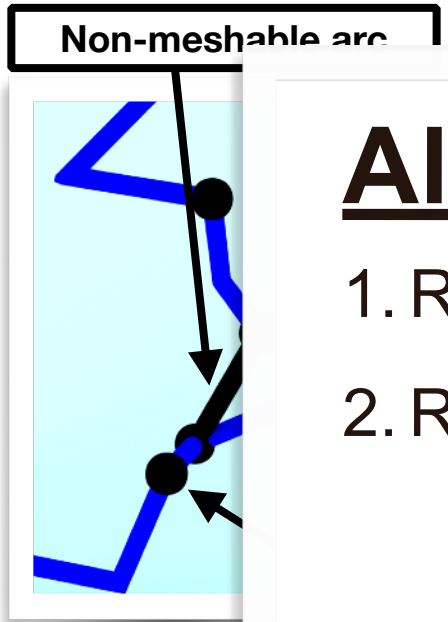


Hex Mesh Singularities





Frame Field Singularities



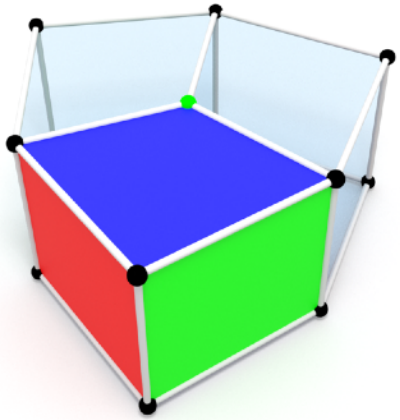
Algorithm

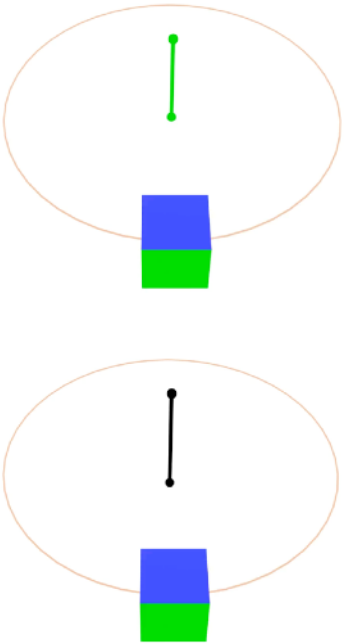
1. Repair non-meshable **arcs**
2. Repair non-meshable **nodes**

Arc Meshability

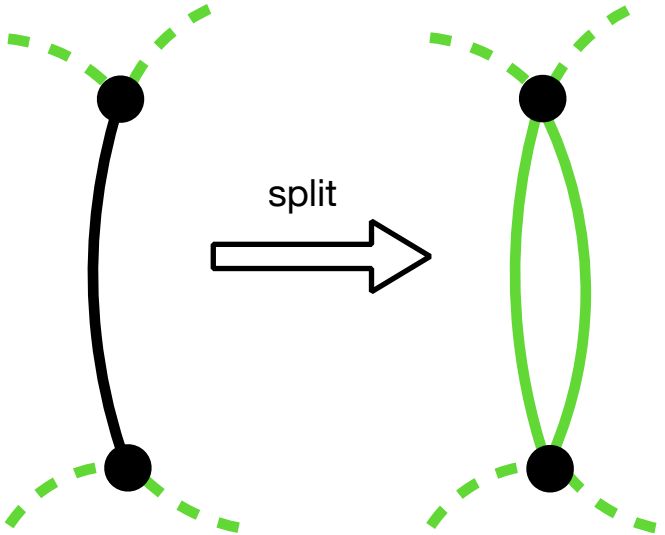
Meshable Monodromy

- Non-meshable Monodromy



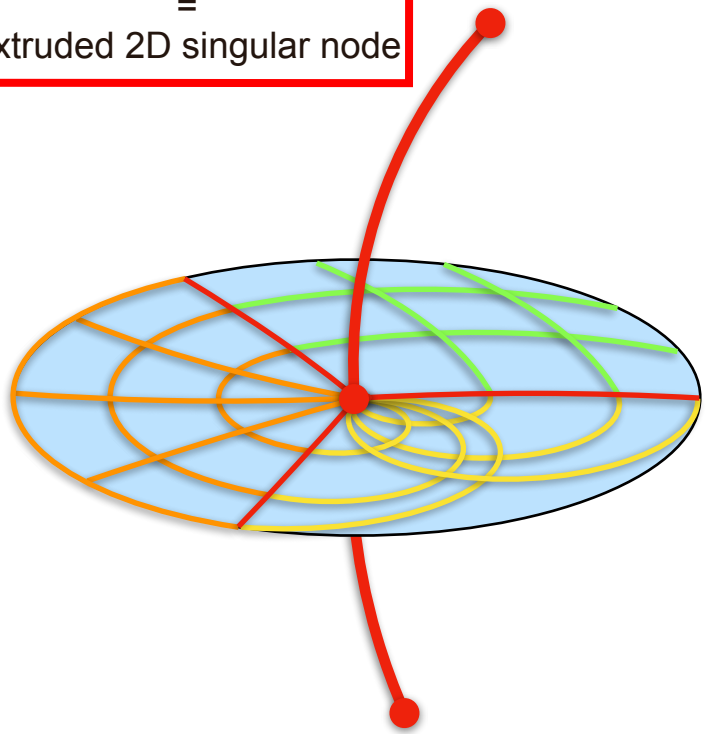


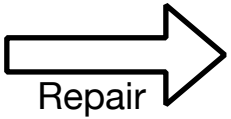
- Repair
  - always splittable into two flow-aligned arcs [Jiang et al. 2014]



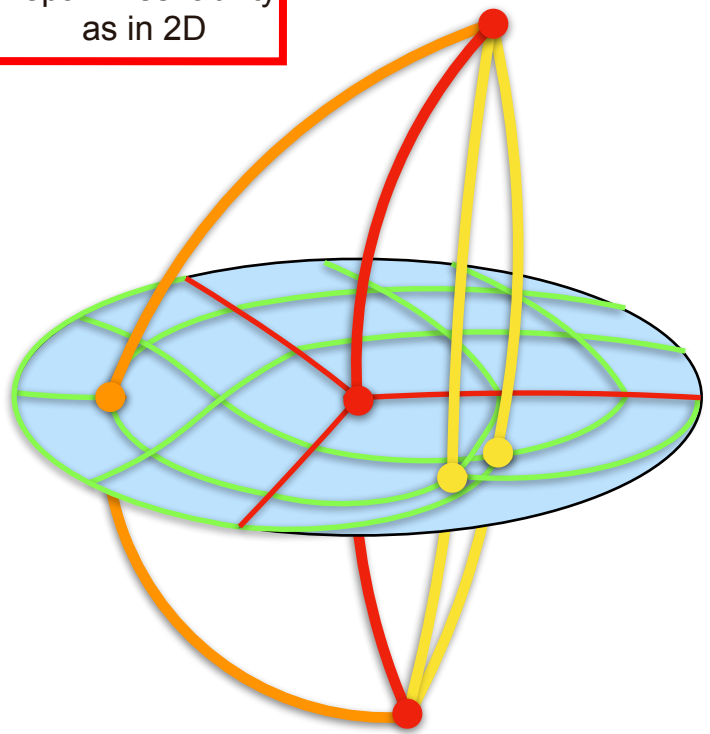
Flow-Aligned Singular Arcs

flow-aligned singular arc
=
extruded 2D singular node



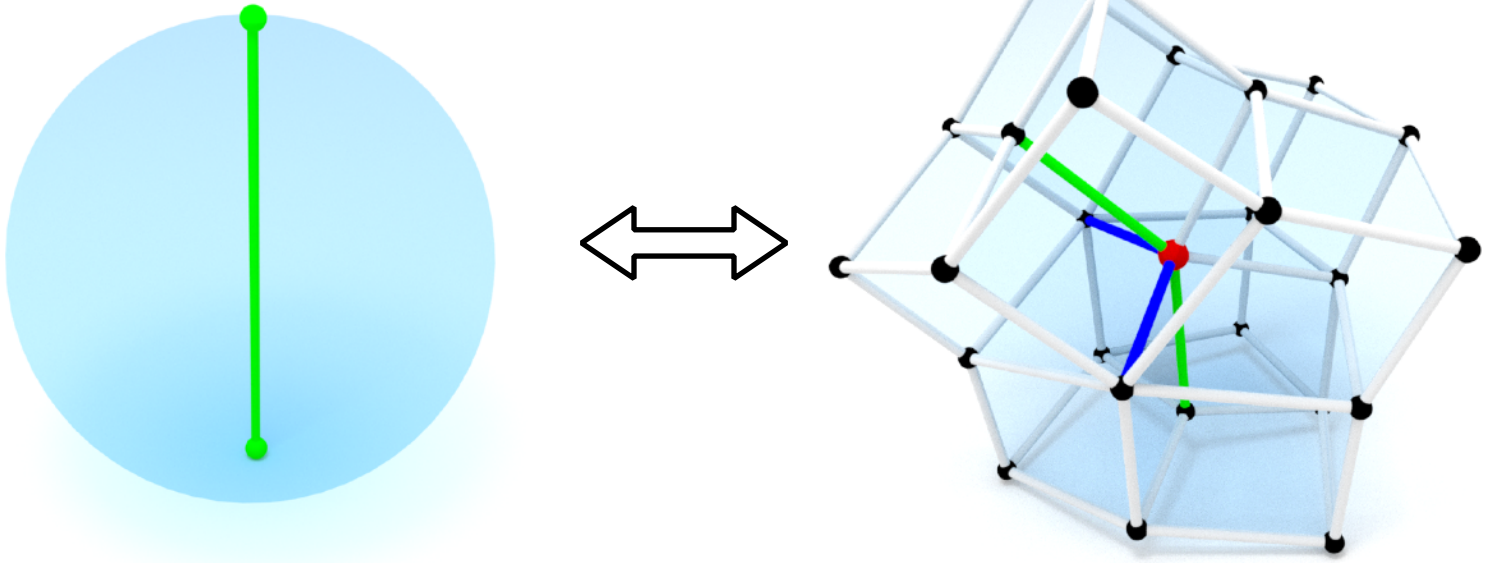


repair meshability
as in 2D

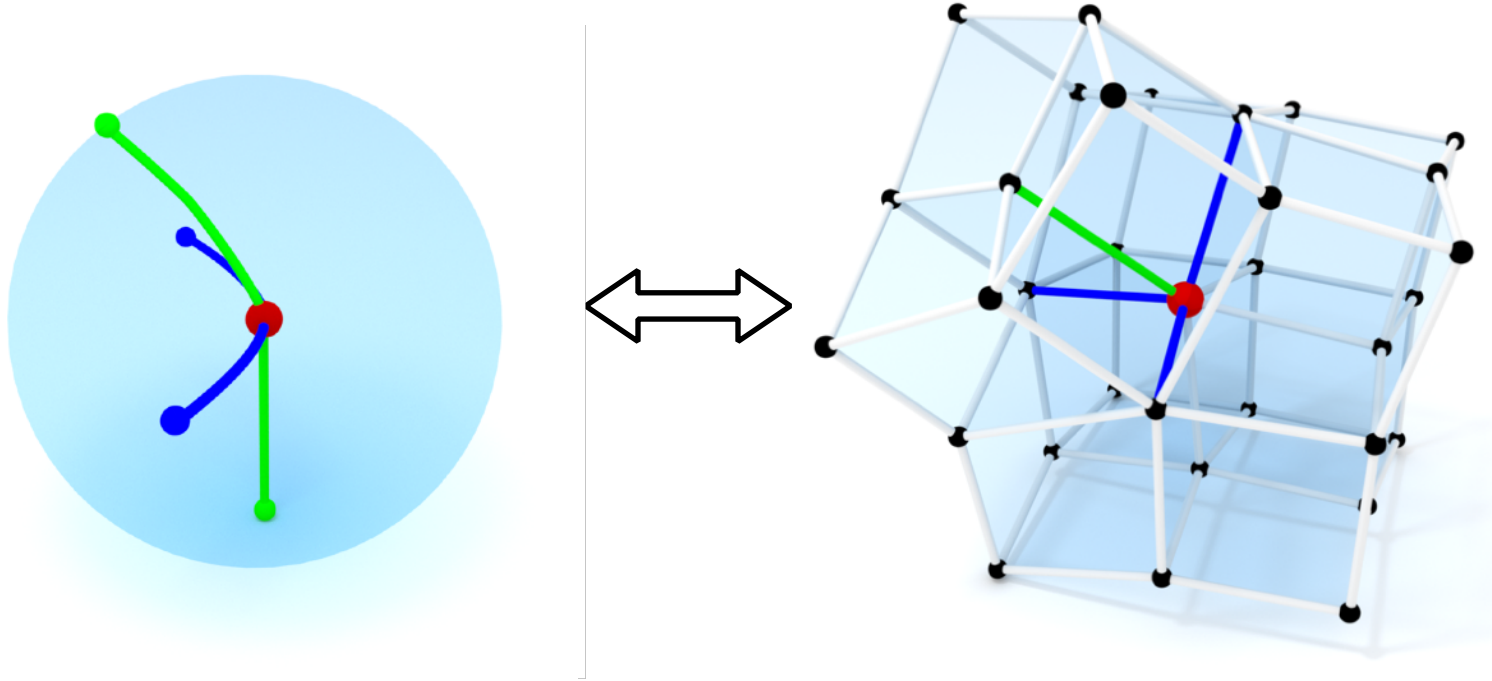


Node Meshability

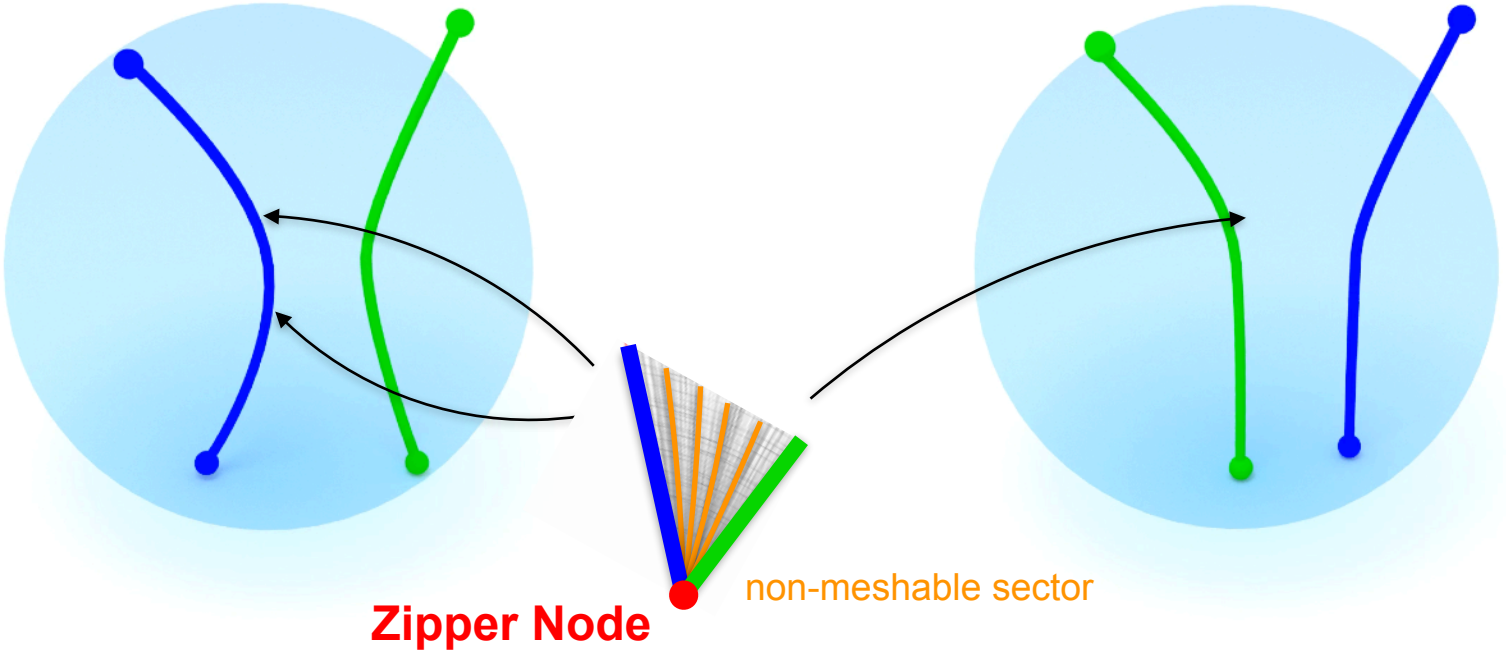
Singular Nodes = Interaction of Singular Arcs



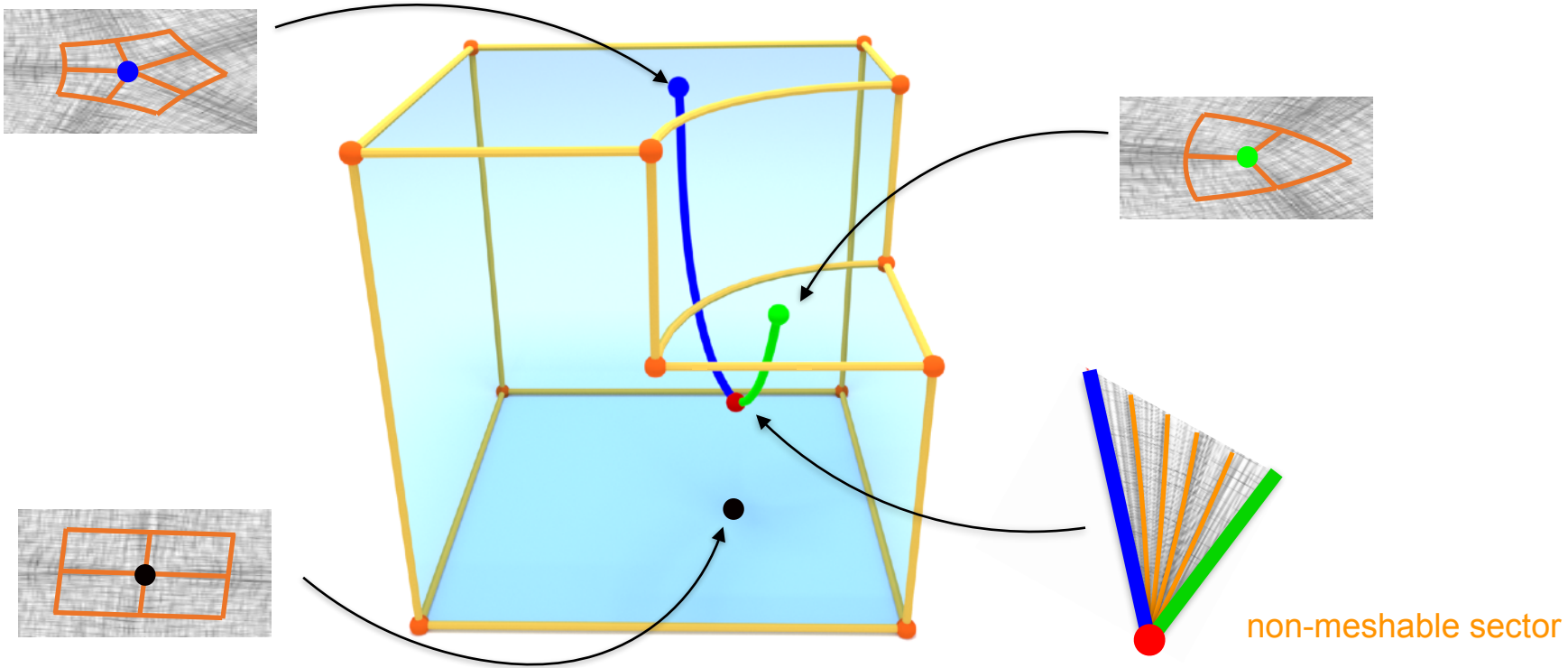
Singular Nodes = Interaction of Singular Arcs



Non-Meshable Interactions

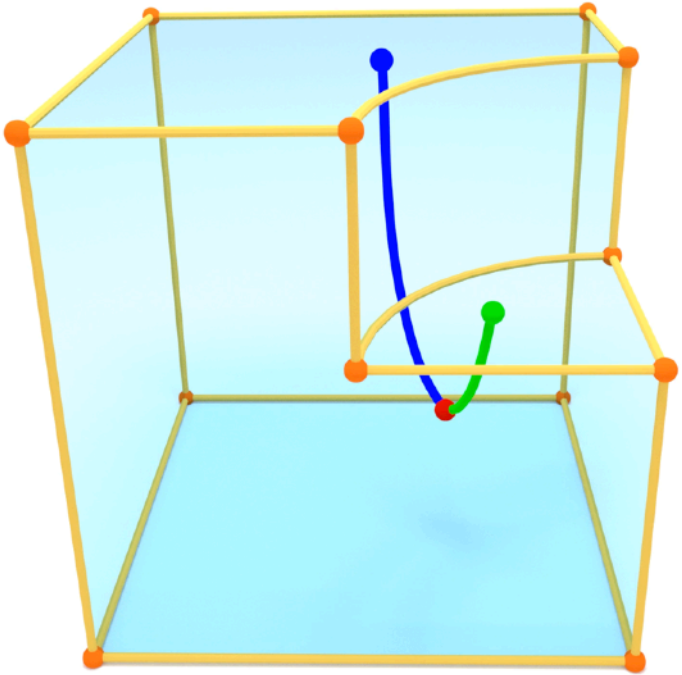


Repairing Zipper Nodes

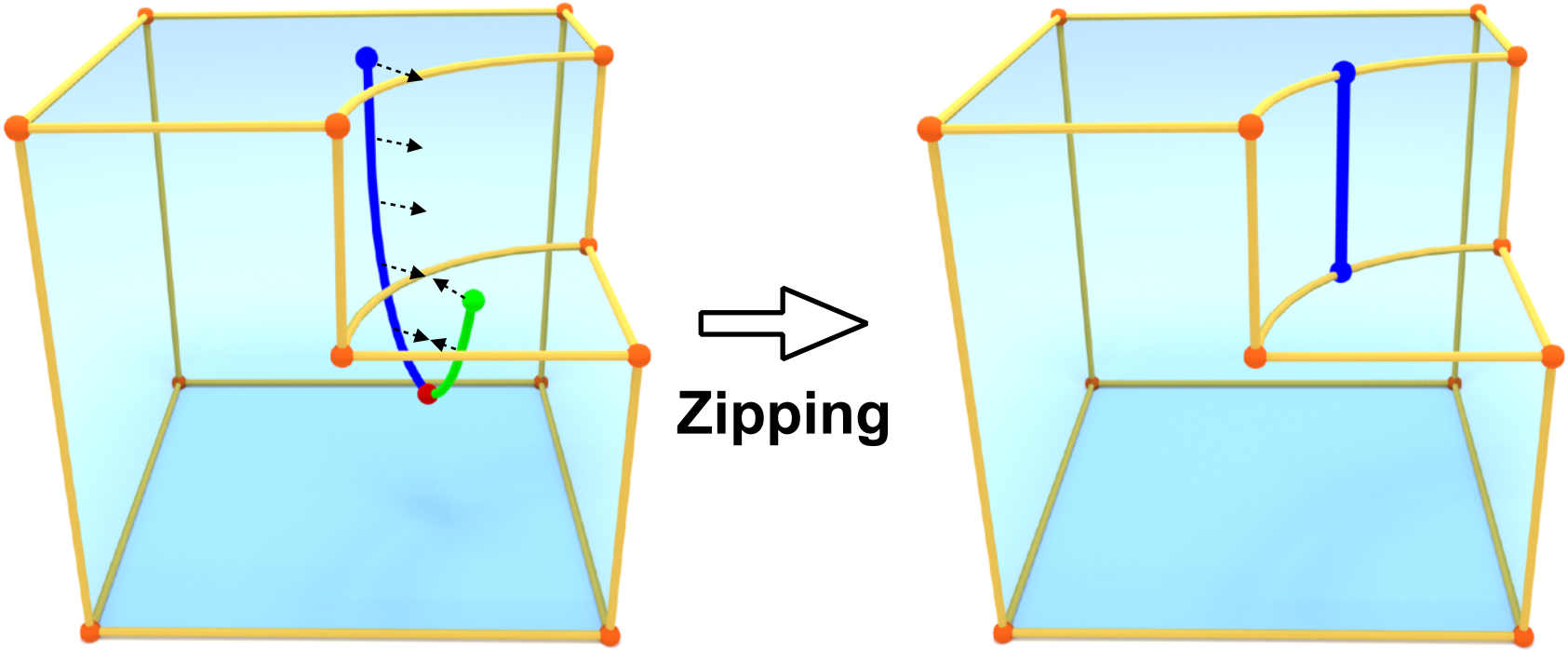


Repairing Zipper Nodes

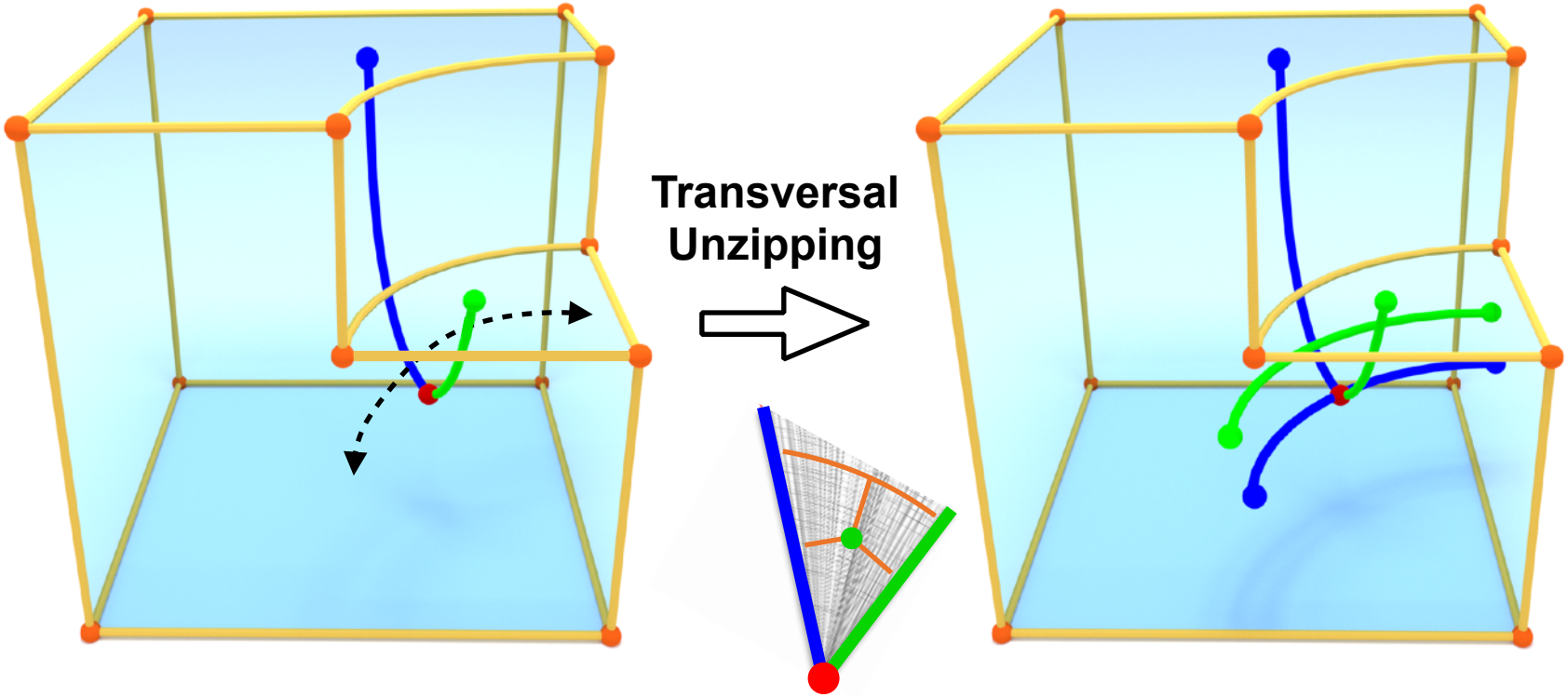
Unzipping



Repairing Zipper Nodes - Strategy II



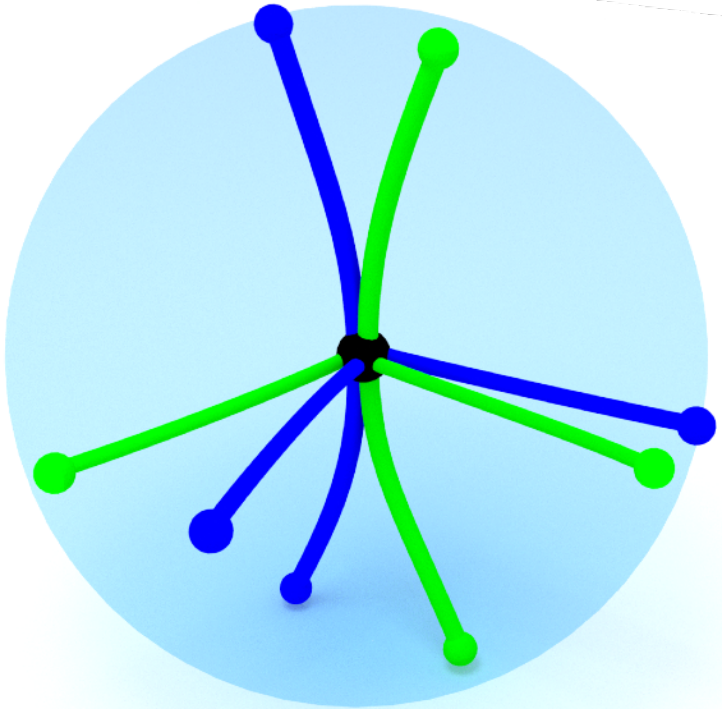
Repairing Zipper Nodes - Strategy III



Repairing General Nodes

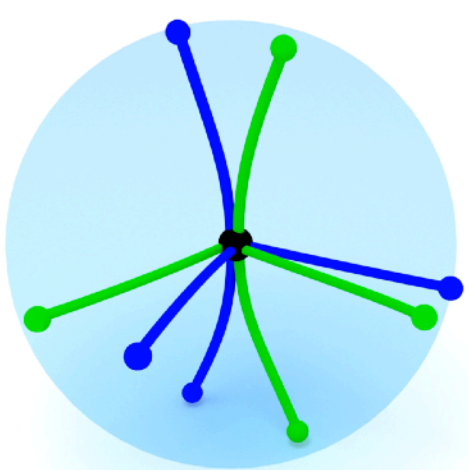
- **Decomposition Theorem**

  Every singular node can locally be decomposed into isolated singular arcs and zipper nodes.

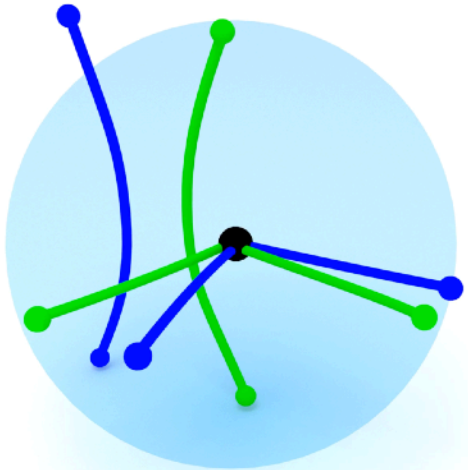


Repairing General Nodes

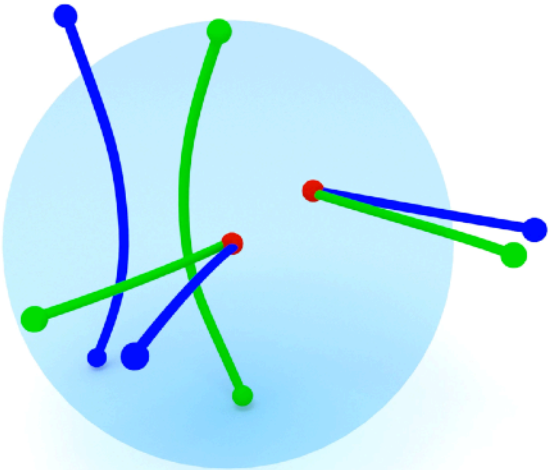
Detaching arcs



Detaching zipper nodes



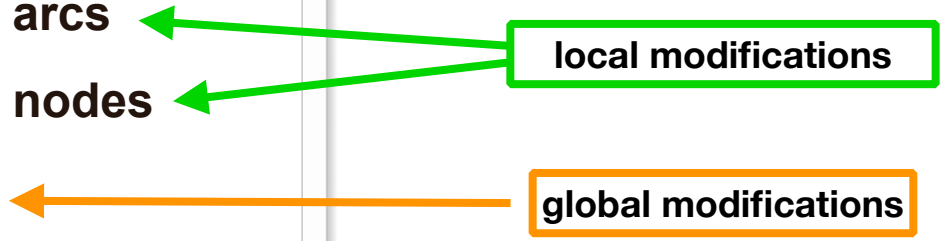
Repairing zipper nodes



Achieving Local Meshability

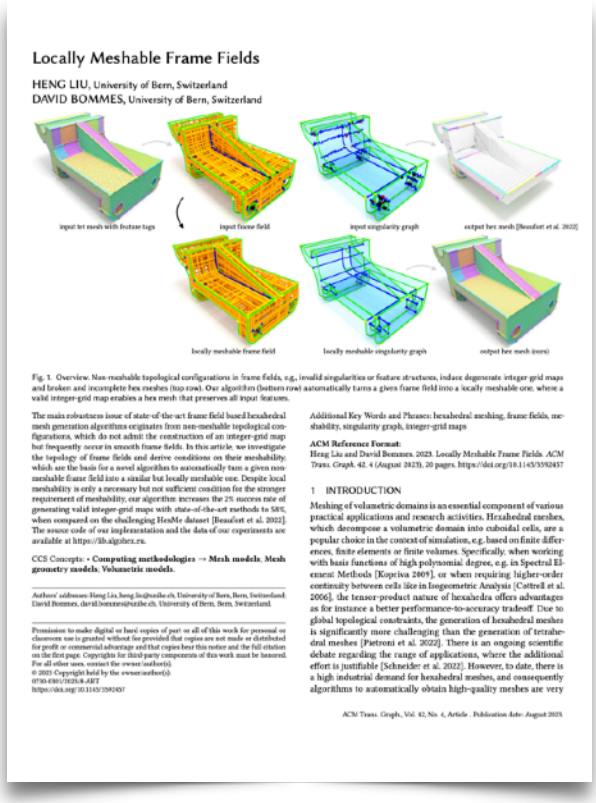
Algorithm

1. Repair non-meshable **arcs**
2. Repair non-meshable **nodes**
3. Repair **zipper nodes**



Many Additional Aspects

- Zipper Node Repair
- Discretization on Tetrahedral Mesh
- Singularity Graph Optimization
- …



Locally Meshable Frame Fields

HENG LIU, University of Bern, Switzerland
DAVID BOMMES, University of Bern, Switzerland

Fig. 1. Overview. Non-meshable topological configurations in frame fields, e.g., invalid singularities or feature structures, induce degenerate integer-grid maps and broken and incomplete hex meshes (top row). Our algorithm (bottom row) automatically turns a given frame field into a locally meshable one, where a valid integer-grid map enables a hex mesh that preserves all input features.

The main robustness issue of state-of-the-art frame field based hexahedral mesh generation algorithms originates from non-meshable topological configurations, which do not admit the construction of an integer-grid map but frequently occur in smooth frame fields. In this article, we investigate the topology of frame fields and derive conditions on their meshability, which are the basis for a novel algorithm to automatically turn a given non-meshable frame field into a similar but locally meshable one. Despite local meshability is only a necessary but not sufficient condition for the stronger requirement of meshability, our algorithm increases the 2% success rate of generating valid integer-grid maps with state-of-the-art methods to 58%, when compared on the challenging HexMe dataset [Beaufort et al. 2022]. The source code of our implementation and the data of our experiments are available at https://lib.algohex.eu.

CCS Concepts: • **Computing methodologies** → **Mesh models**; **Mesh geometry models**; **Volumetric models**.

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https://doi.org/10.1145/3592457

Additional Key Words and Phrases: hexahedral meshing, frame fields, meshability, singularity graph, integer-grid maps

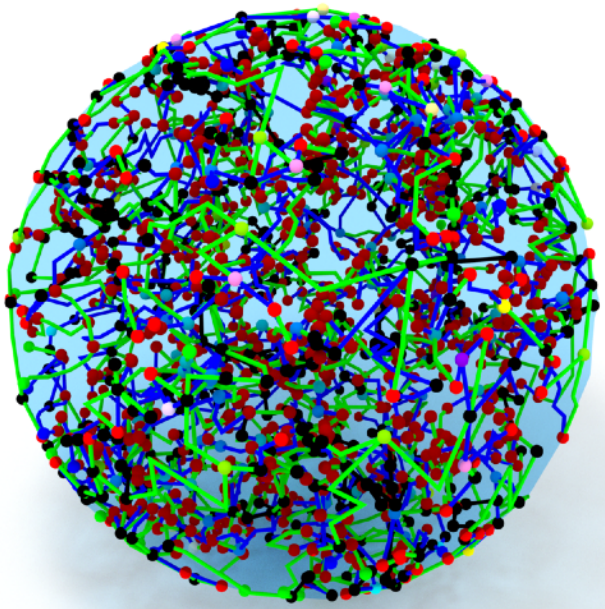
ACM Reference Format:
Heng Liu and David Bommes. 2023. Locally Meshable Frame Fields. *ACM Trans. Graph.* 42, 4 (August 2023), 20 pages. https://doi.org/10.1145/3592457

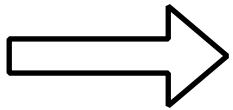
1 INTRODUCTION

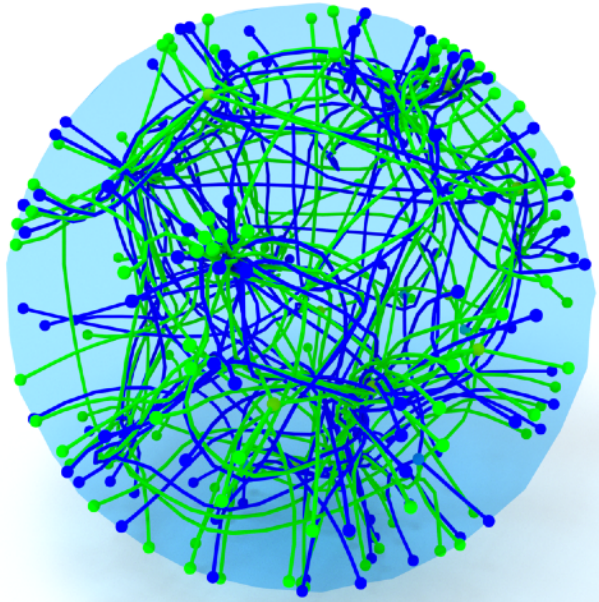
Meshing of volumetric domains is an essential component of various practical applications and research activities. Hexahedral meshes, which decompose a volumetric domain into cuboidal cells, are a popular choice in the context of simulation, e.g. based on finite differences, finite elements or finite volumes. Specifically, when working with basis functions of high polynomial degree, e.g. in Spectral Element Methods [Kopriva 2009], or when requiring higher-order continuity between cells like in Isogeometric Analysis [Cottrell et al. 2006], the tensor-product nature of hexahedra offers advantages as for instance a better performance-to-accuracy tradeoff. Due to global topological constraints, the generation of hexahedral meshes is significantly more challenging than the generation of tetrahedral meshes [Pietroni et al. 2022]. There is an ongoing scientific debate regarding the range of applications, where the additional effort is justifiable [Schneider et al. 2022]. However, to date, there is a high industrial demand for hexahedral meshes, and consequently algorithms to automatically obtain high-quality meshes are very

Results

Stress Test





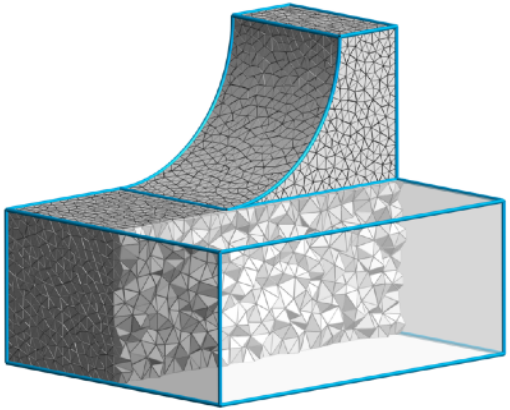


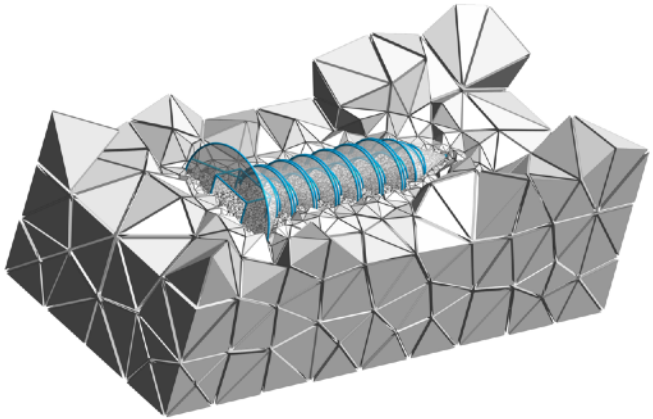
input

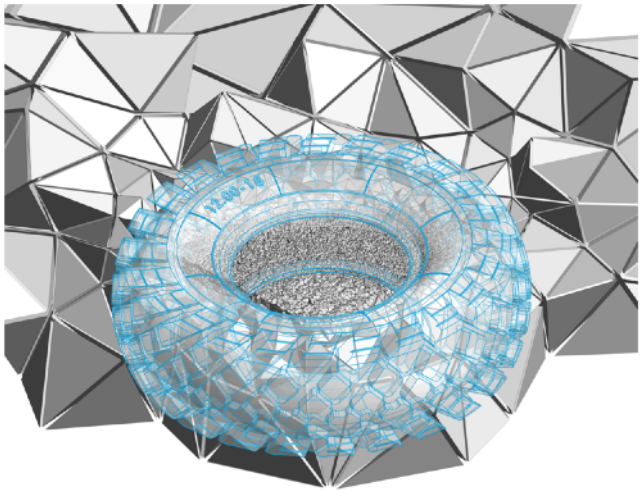
locally meshable

Evaluation

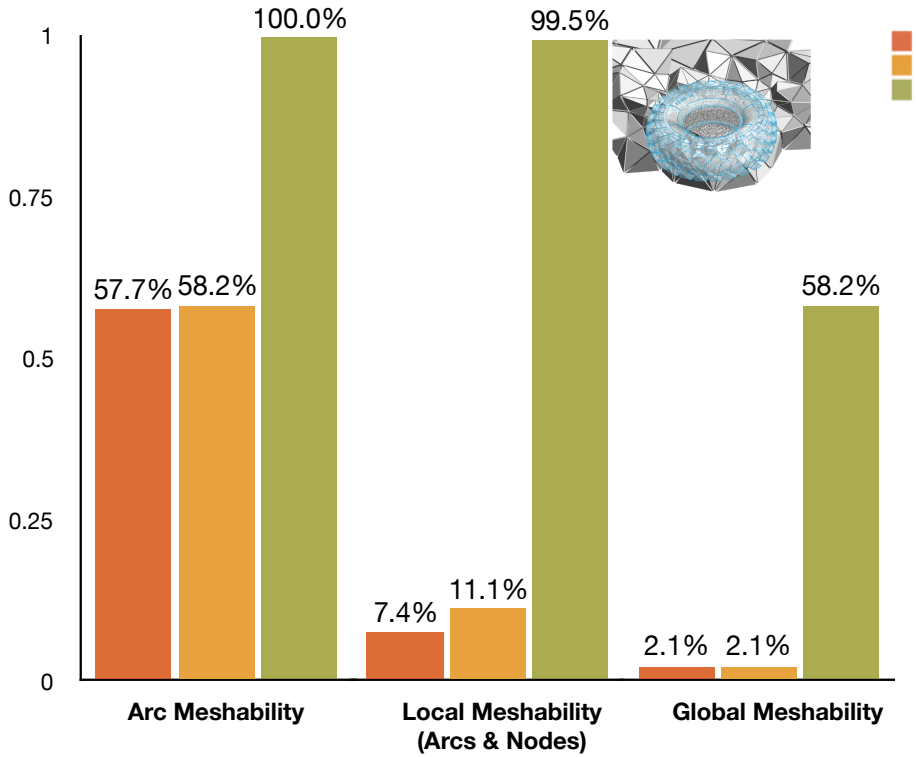
- **HEXME - Dataset** [Beaufort et al. 2022]
  - collection of **189 domains** to evaluate and challenge hexmeshers
    - categories: simple, nasty, industrial, including multi-material
  - **goal:** facilitate comparison & guide future research



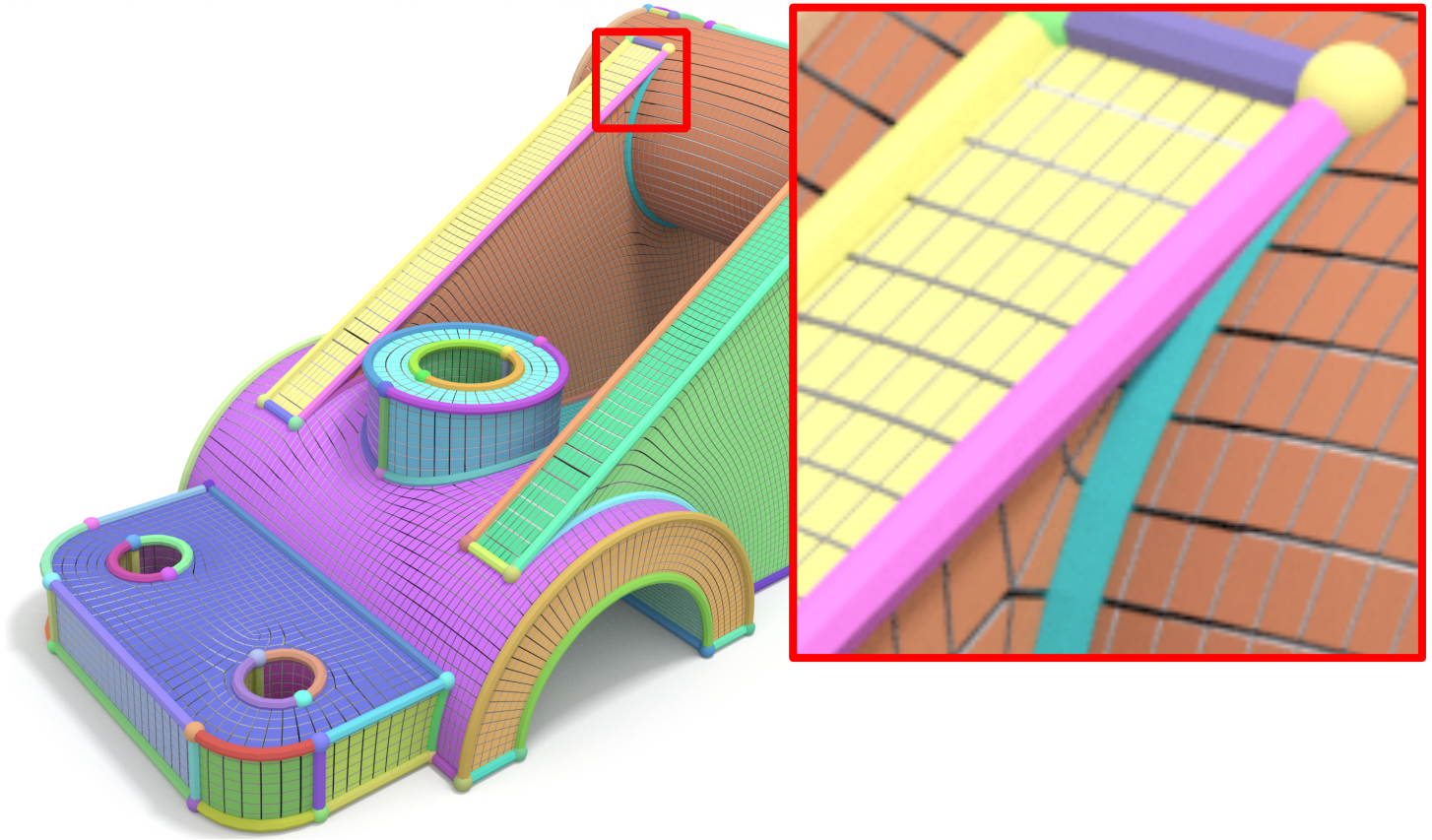




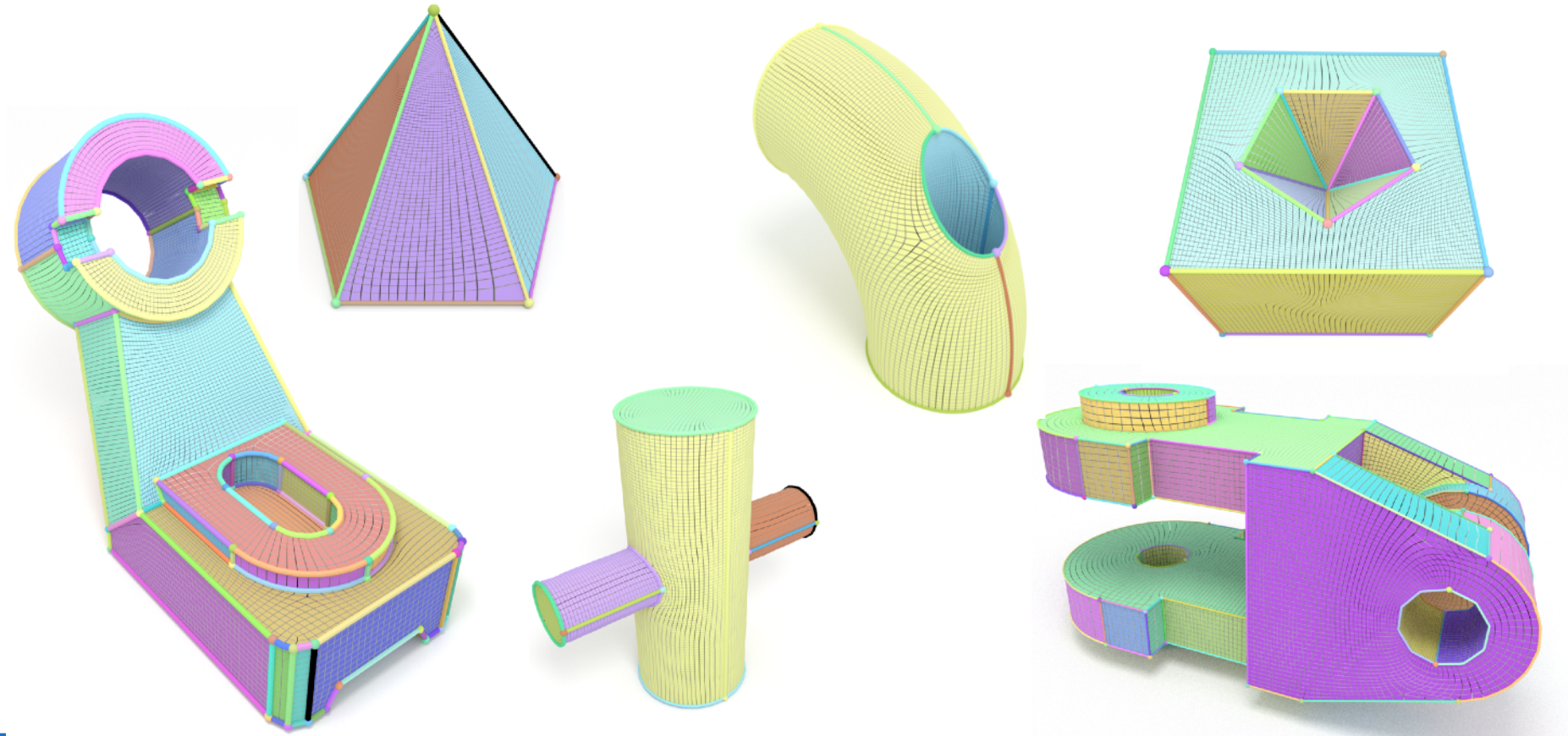
Evaluation


1
0.75
0.5
0.25
0
100.0%
99.5%
57.7%
58.2%
58.2%
7.4%
11.1%
2.1%
2.1%
Arc Meshability
Local Meshability
(Arcs & Nodes)
Global Meshability
All-hex meshing using singularity-restricted field [Li et al . 2012]
Frame Field Singularity Correction for Automatic Hexahedralization [Jiang et al. 2014]
Ours

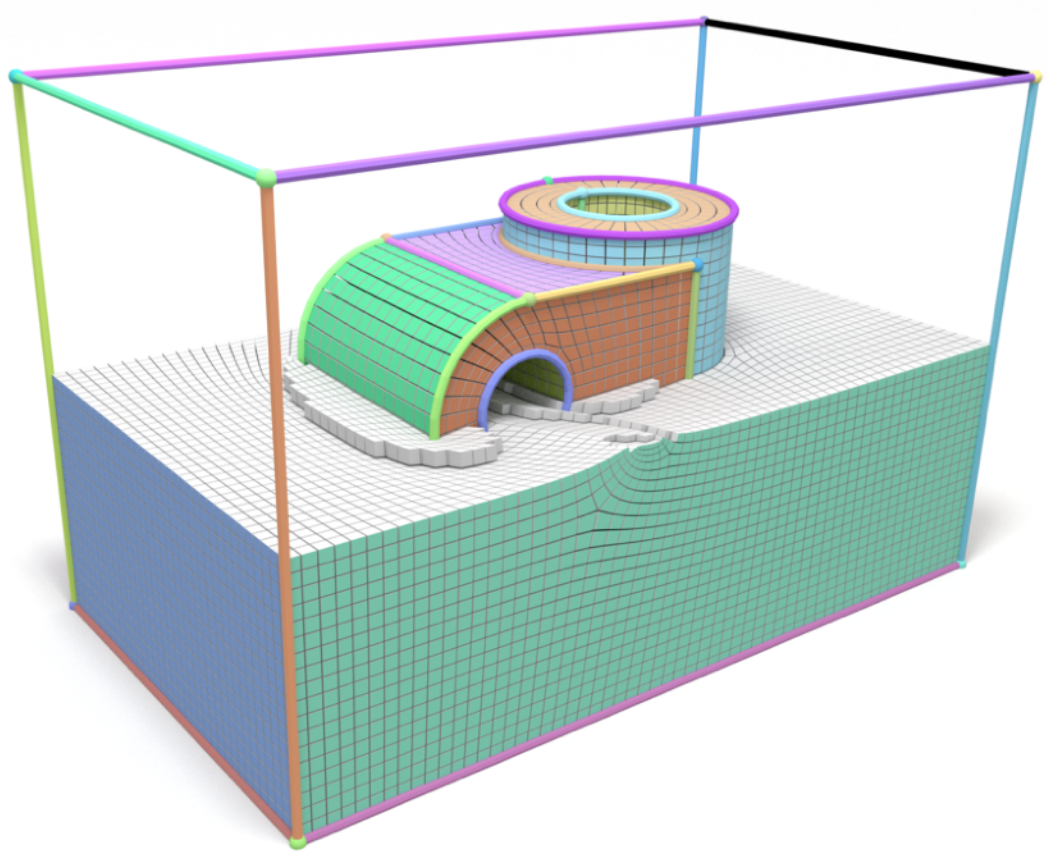
Evaluation — Hex Meshes



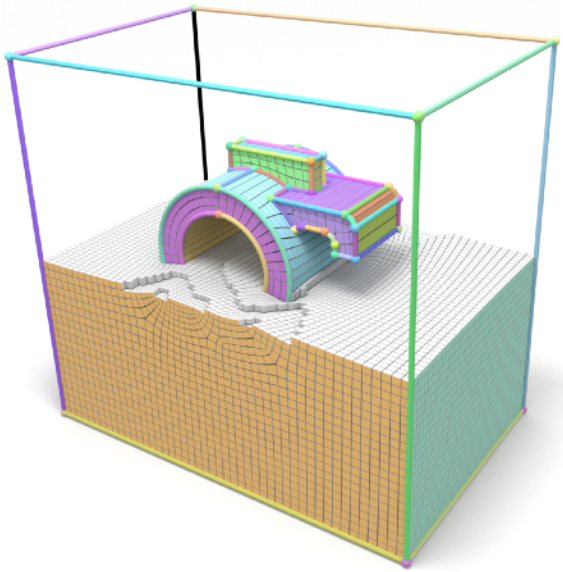
Evaluation — Hex Meshes

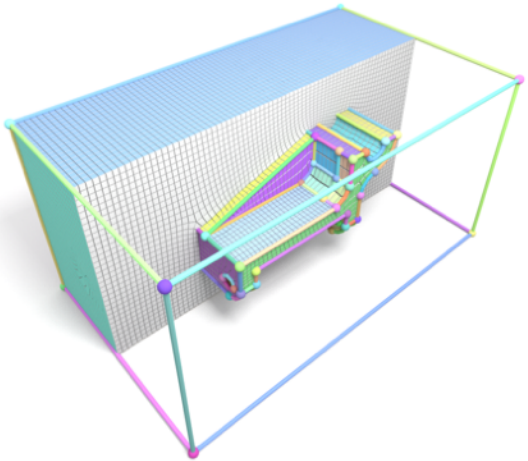


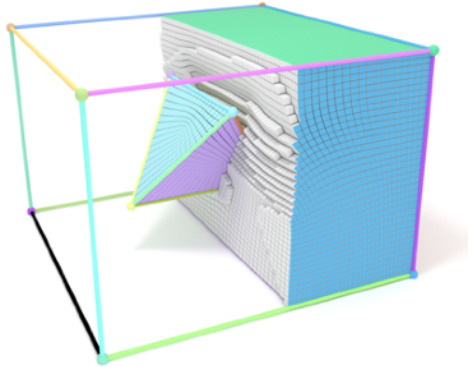
Evaluation — Hex Meshes

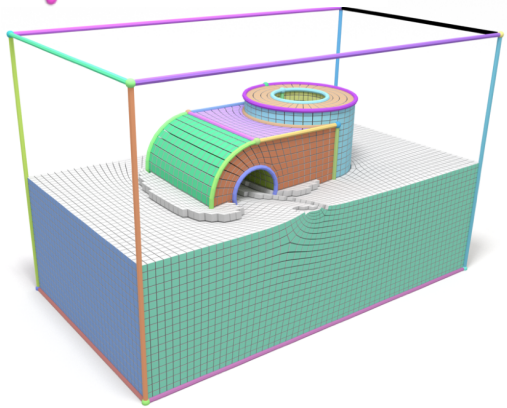


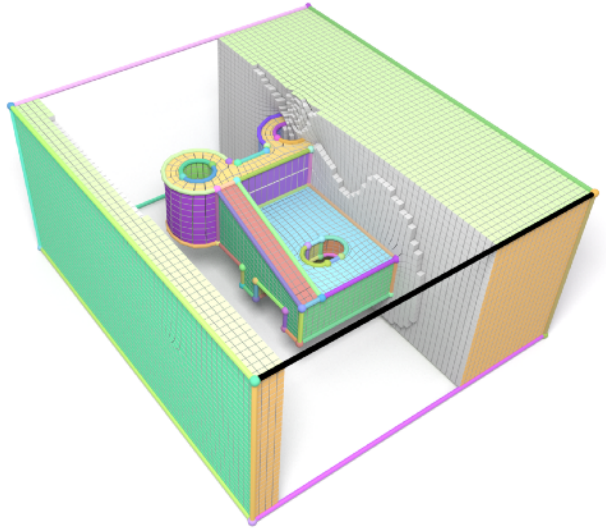
Evaluation — Hex Meshes



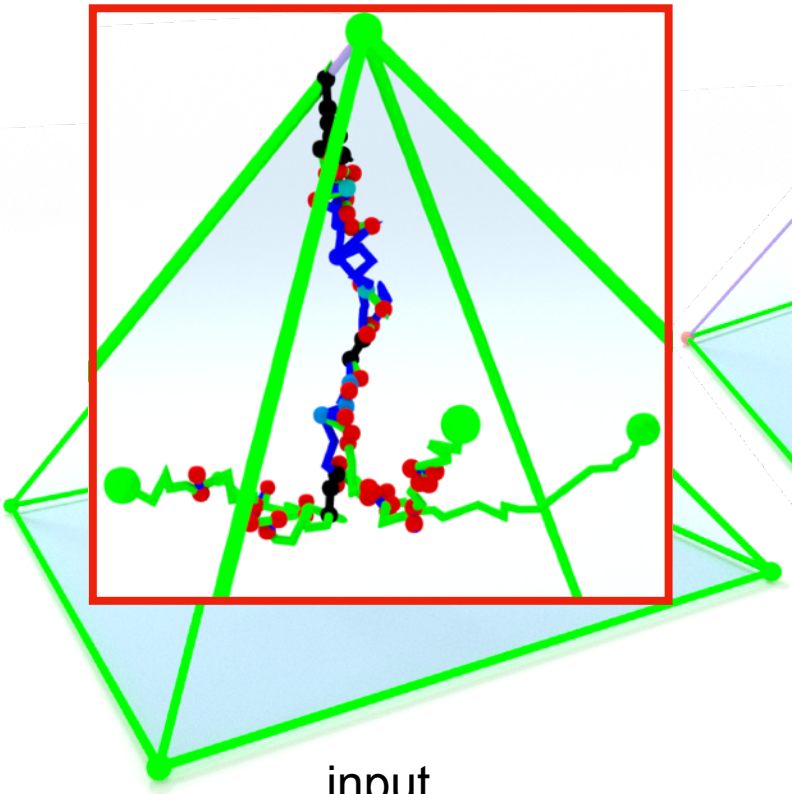








Pyramid



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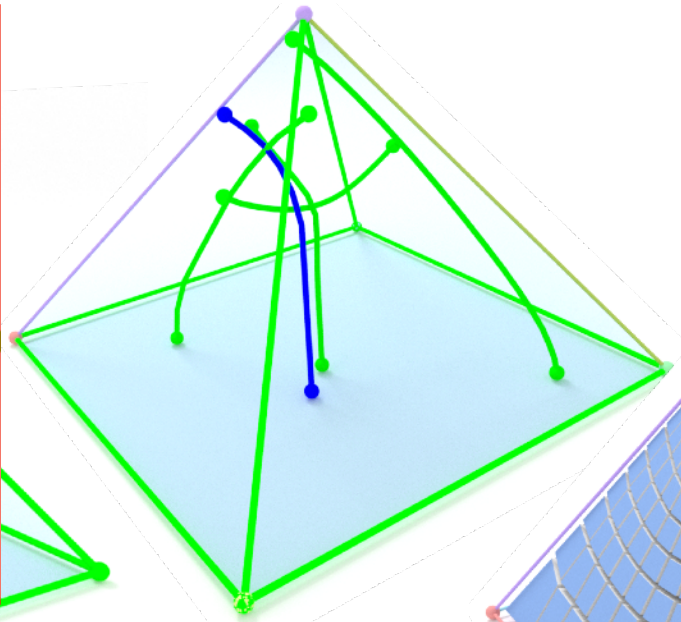
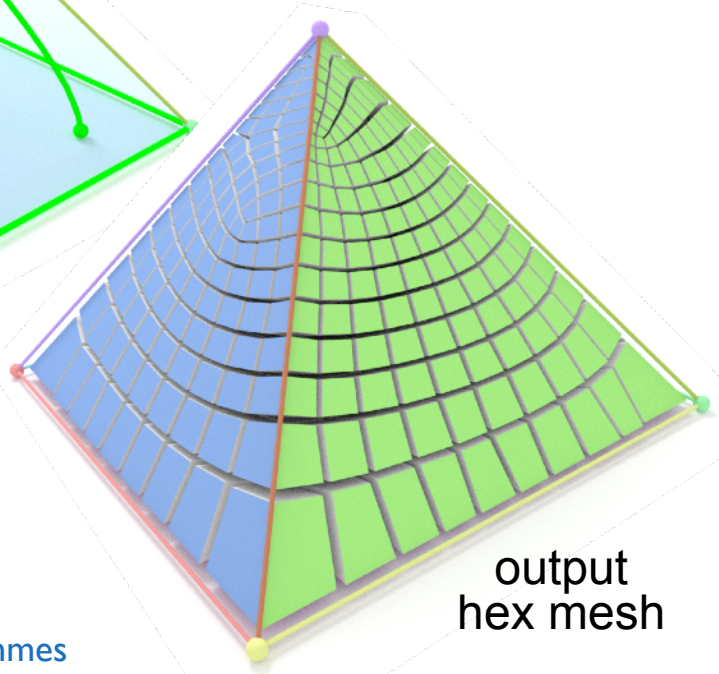
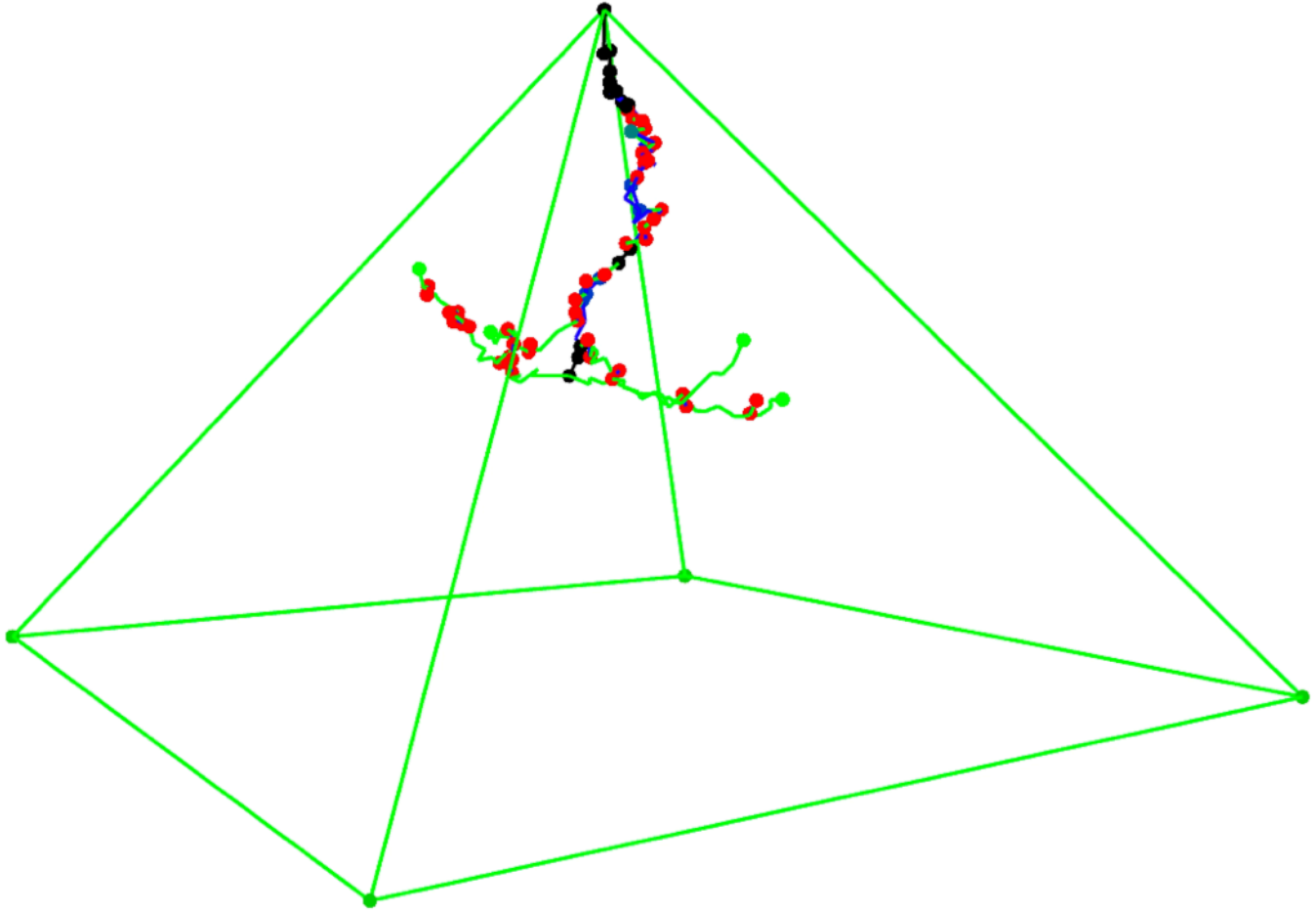


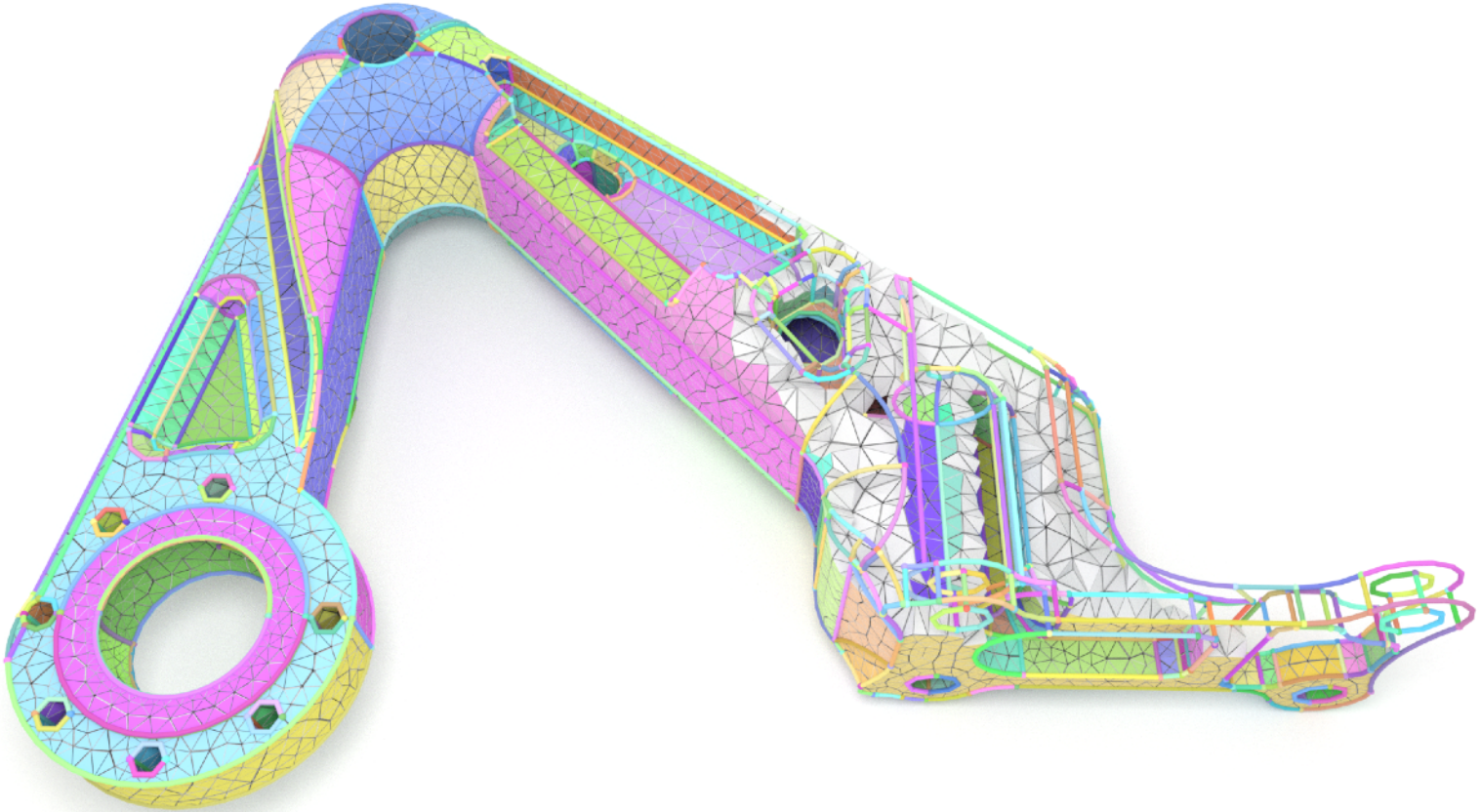
locally meshable



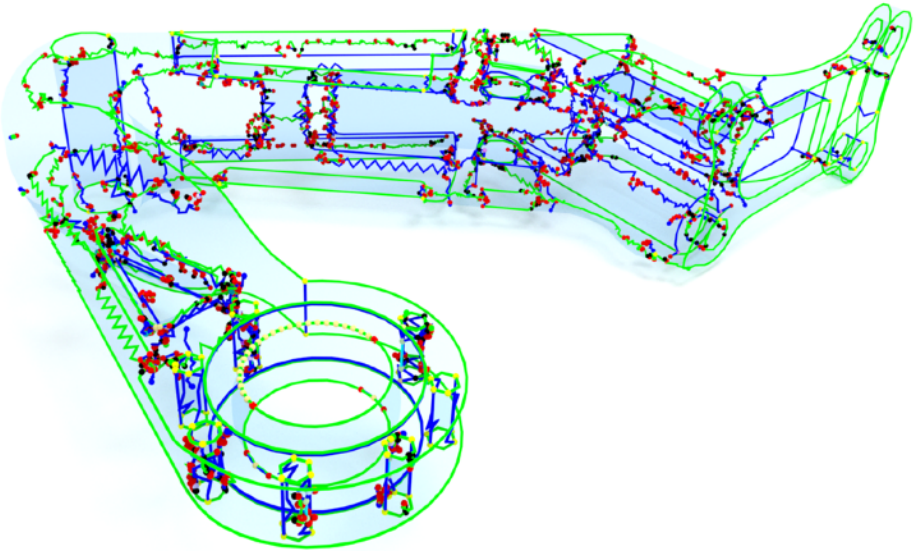
output hex mesh



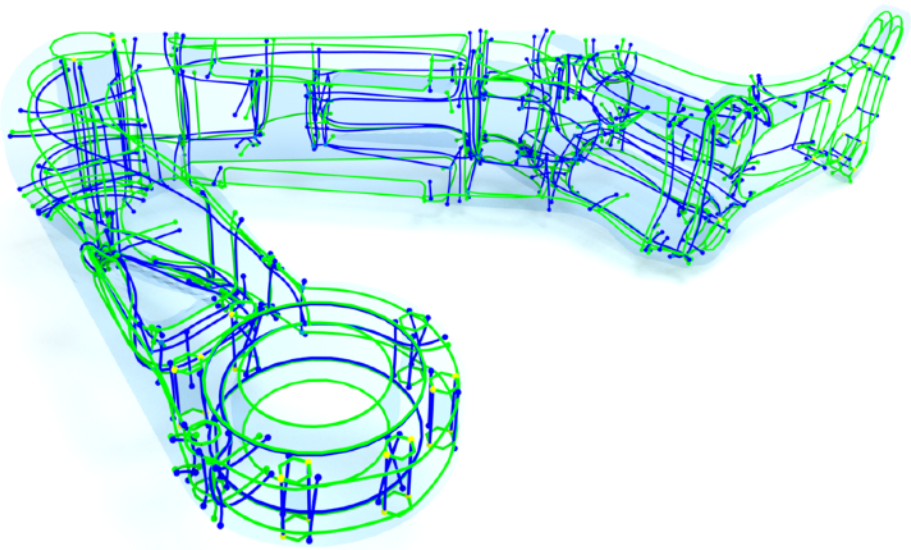
HexMe 129



HexMe 129



input



locally meshable

Code

- **libAlgoHex — http://lib.algohex.eu**
  - C++, open source
  - **goal:** facilitate research & comparison
  - complete IGM based hex meshing pipeline
    - locally meshable frame-fields
    - IGM parametrization
    - mesh extraction

- **Part of ERC project AlgoHex**
  - http://www.algohex.eu
  - stay tuned!

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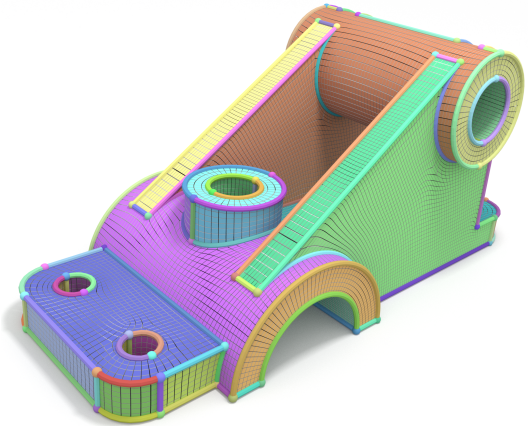
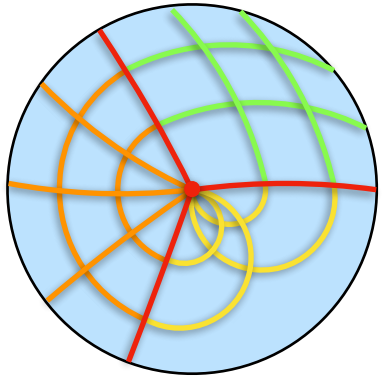
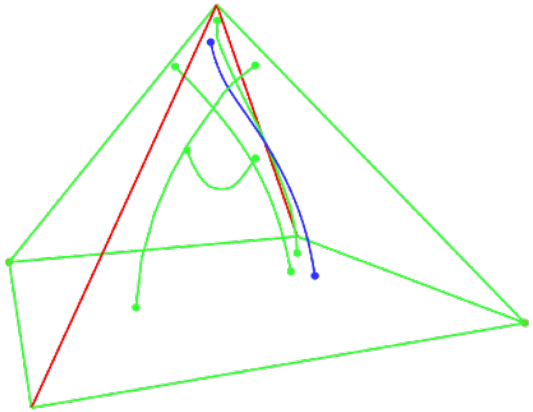
Summary & Outlook

- **Integer-Grid Maps**
  - offer good balance between quality criteria
  - efficiency through frame fields
- **Quadrilateral Meshing**
  - robustness / performance / quality / control
  - complete toolbox available
- **Hexahedral Meshing**
  - significantly more challenging, many unsolved aspects (e.g. singular topology, robust volumetric maps)
  - locally meshable frame fields
  - important next step towards robustness:

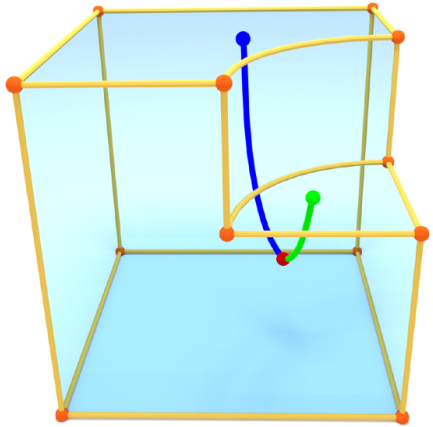
    ➡ globally meshable frame fields

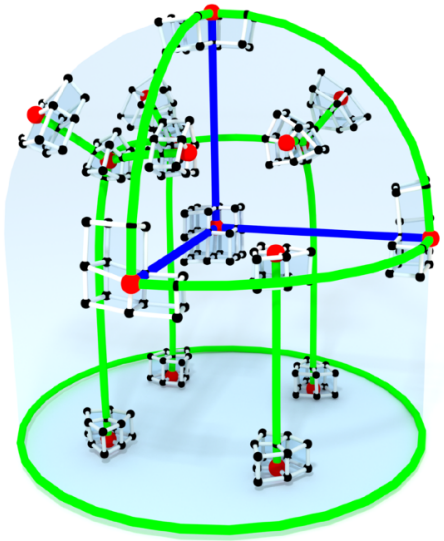






Thank You!





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