Reconstruction of Concise Meshes from 3D Point Clouds

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Joint work with…

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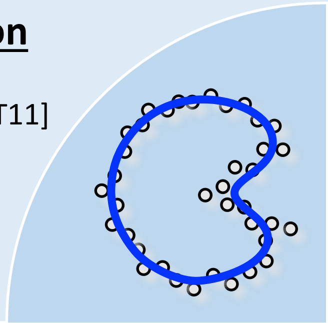
Project:
tong-zhao.github.io/vsr

Surface Reconstruction

Smoothness-based reconstruction

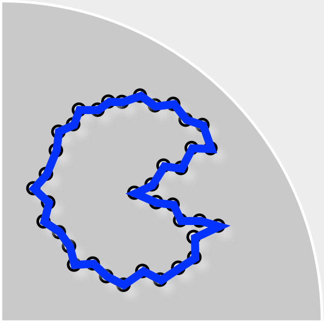
MLODS[MLR+22]
RFEPS[XWD+22]
Iterative Poisson[HWW+22a]
Differentiable Poisson[PJL+21]
FSSR[FG14]
VIPSS[HCJ19]
Screened Poisson [KBH13]
Structured Point Set[LA13]

APSS[GG07]
Spectral[ACTD07]
SSD[CT11]
RIMLS[OGG09]
IMLS[SOS05]
RMLS[FCOS05]
Poisson[KBH06]
PSS[ABCO+01]
MPU[OBA+03a]



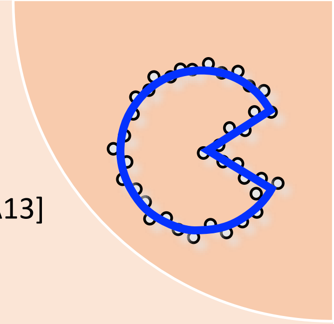
Interpolation-based reconstruction

Singular Cocone[DW13]
Robust Cocone[DG04]
Advancing Front[SSFS06]



Power Crust[ACK+01]
Tight Cocone[DS03]
Crust[ACD+00]

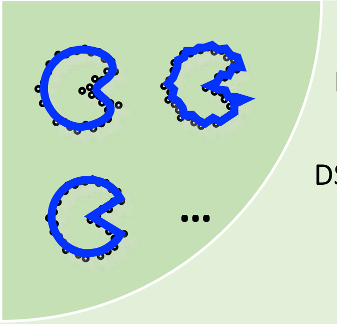
Primitive-based reconstruction



GoCoPP[YL22]
Kinetic[BL20a]
Polyfit[NW17]
Structured Point Set[LA13]

Efficient RANSAC[SDK09]

Learning-based reconstruction

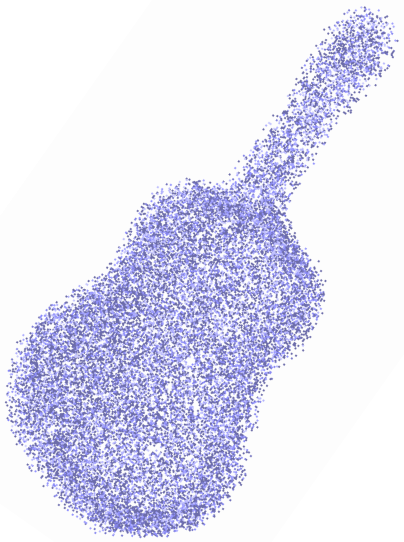


POCO[BM22]
Neural Dual Contouring[CTFZ22]
Neural Marching Cube[CZ21]
DSE[RGA+21]
Minkovski Engine[CGS19]

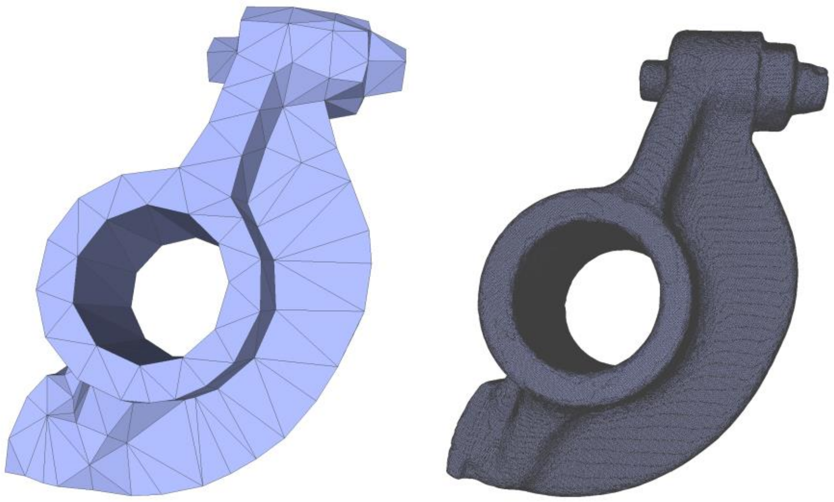
O-CNN[WLG+17]
OGN[TDB17]
AtlasNet[GFK+18]
DeepSDF[PFS+19]
Point2Mesh[HMGC20]

Challenges

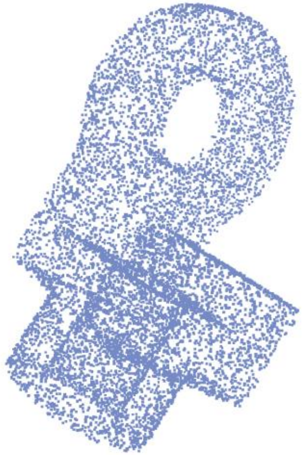
Missing data



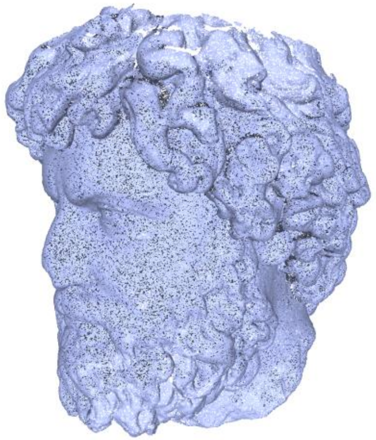
Noise



Complexity-distortion tradeoff



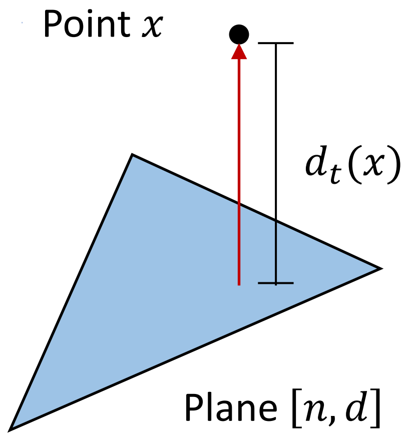
Sharp features



Multi-scale features

...

Quadric Error Metric



Squared distance to plane

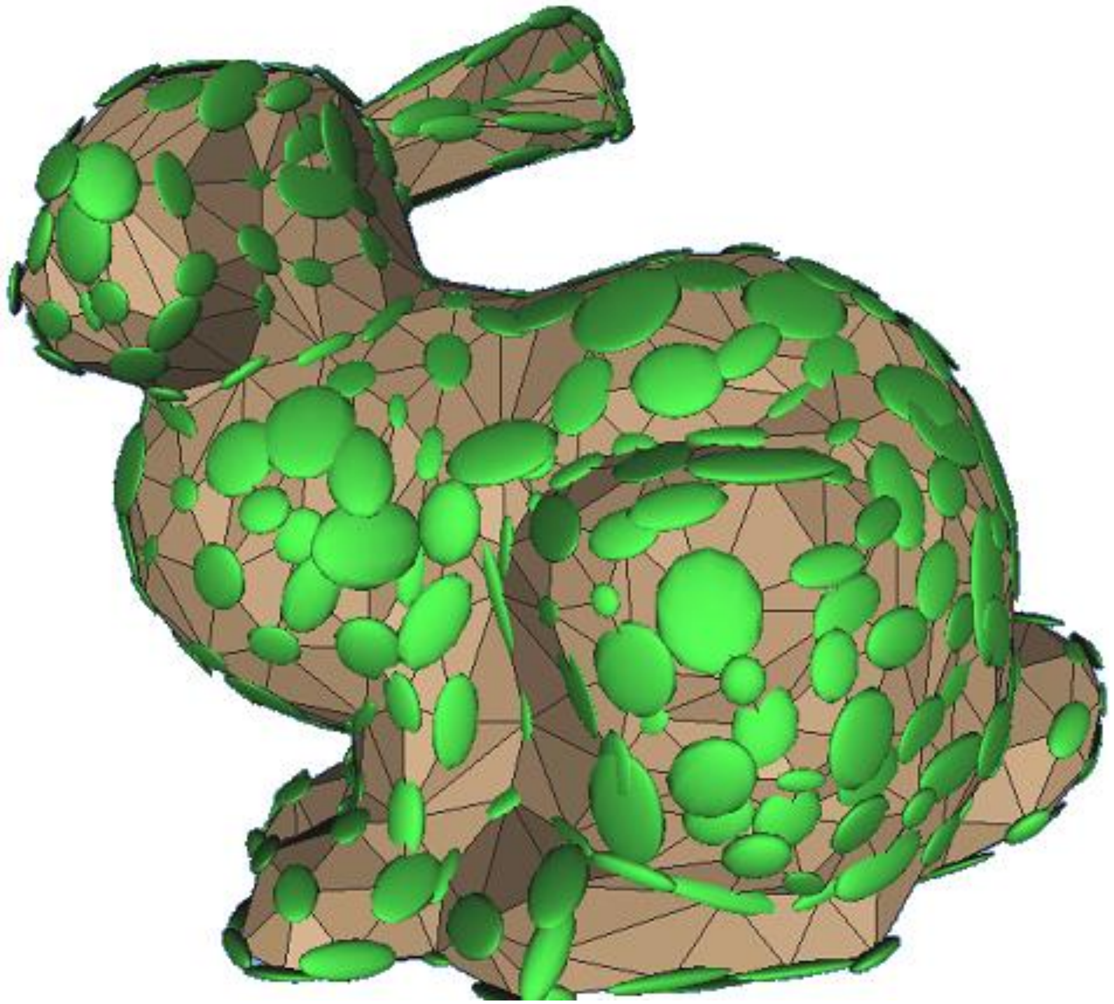
$$p = (x, y, z, 1)^T, \quad q = (a, b, c, d)^T$$

$$dist(q, p)^2 = (q^T p)^2 = p^T (q q^T) p =: p^T Q_q p$$

$$Q_q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & b^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

[Garland, Heckbert 97]

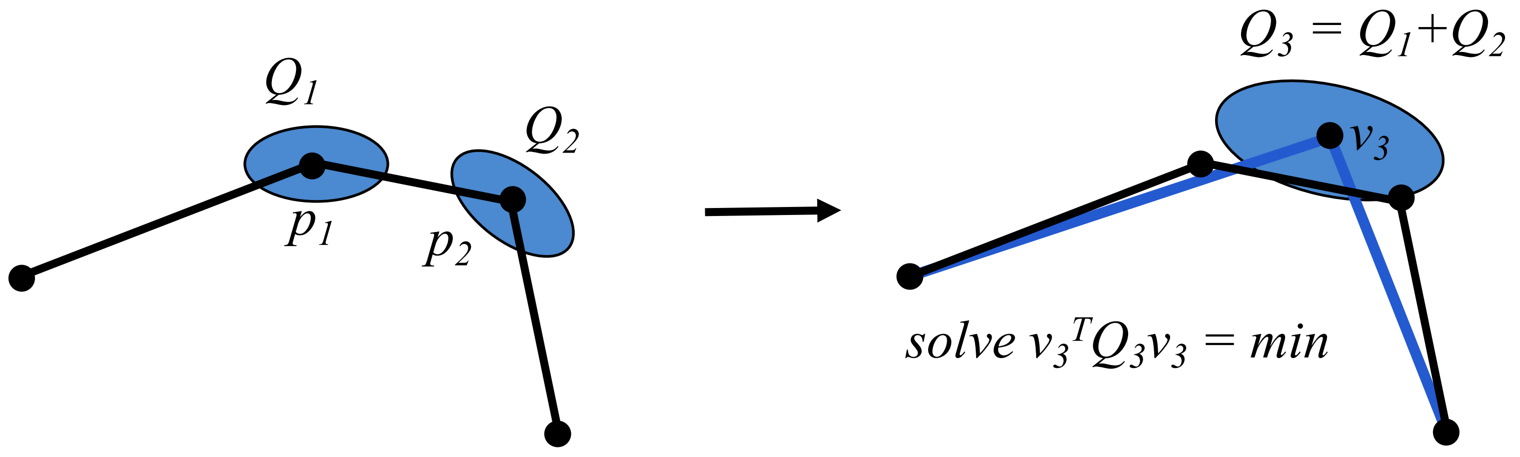
Quadric Error Metrics

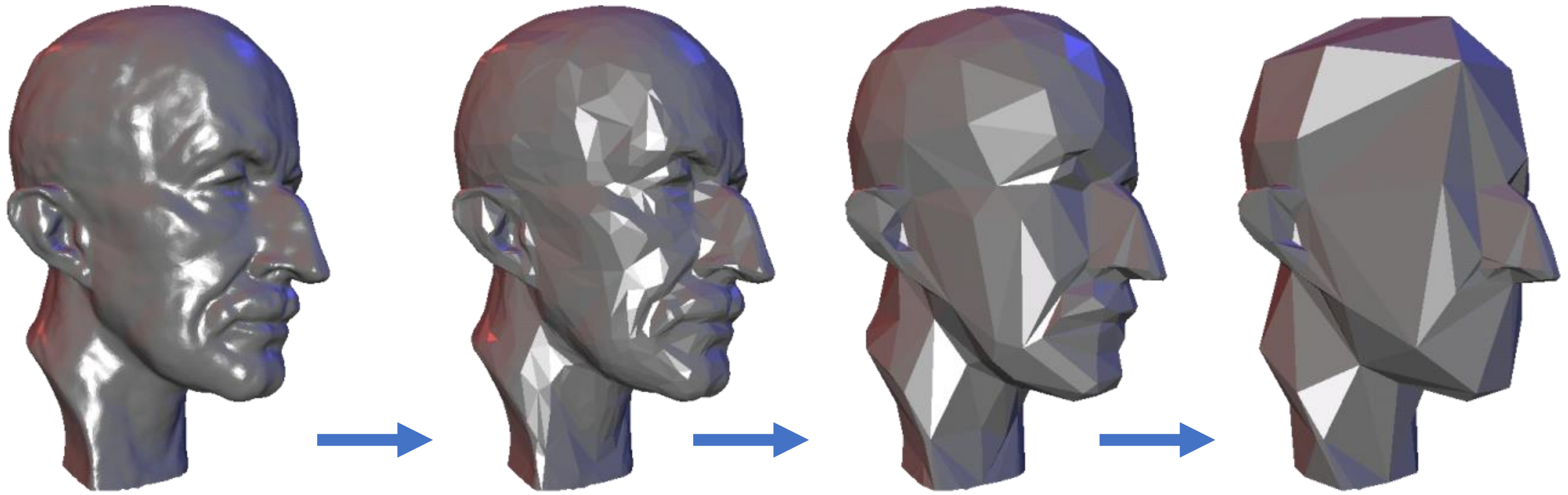


Ellipsoid = error isolevel

[Garland, Heckbert 97]

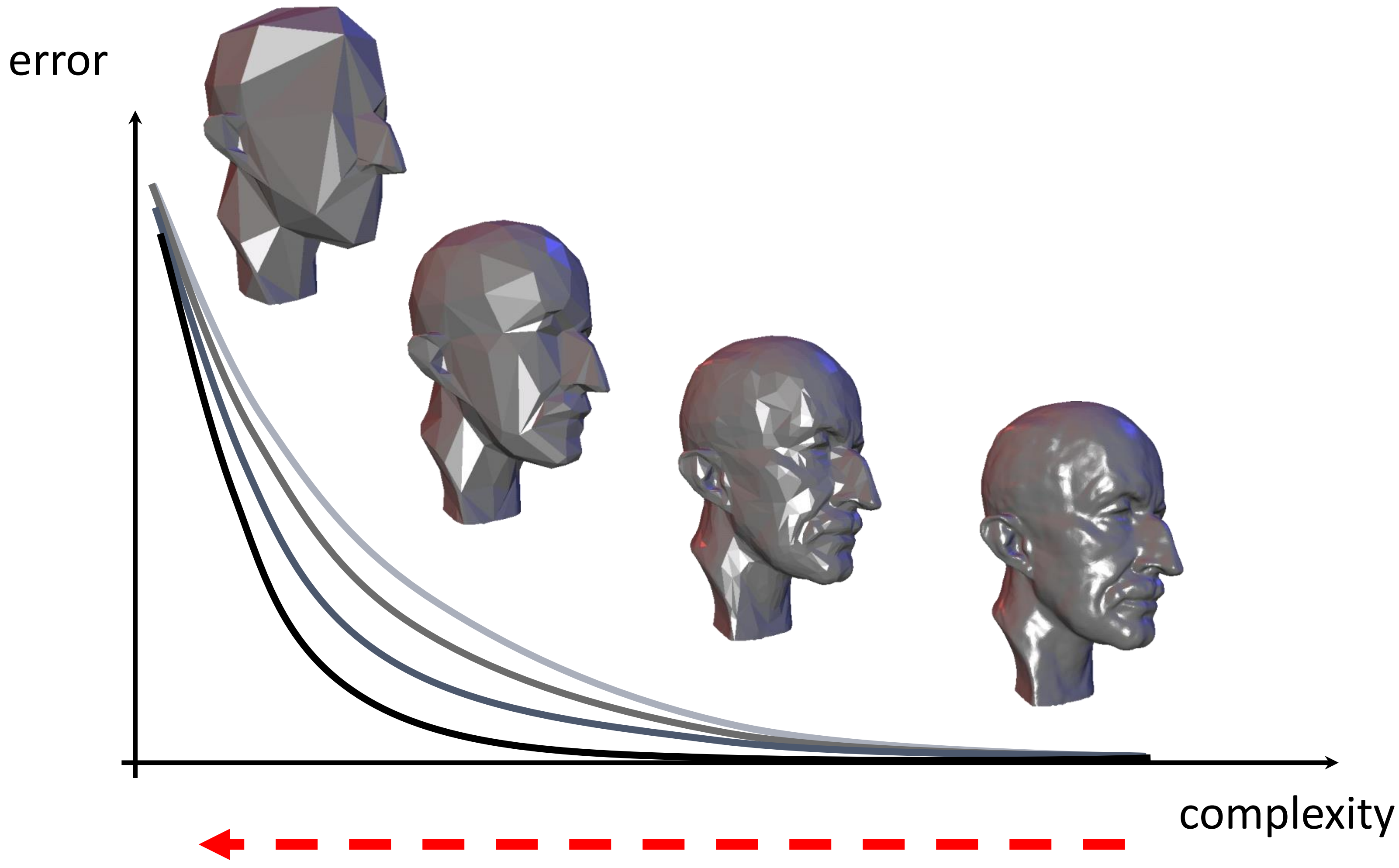
Quadric Error Metrics for Mesh Decimation



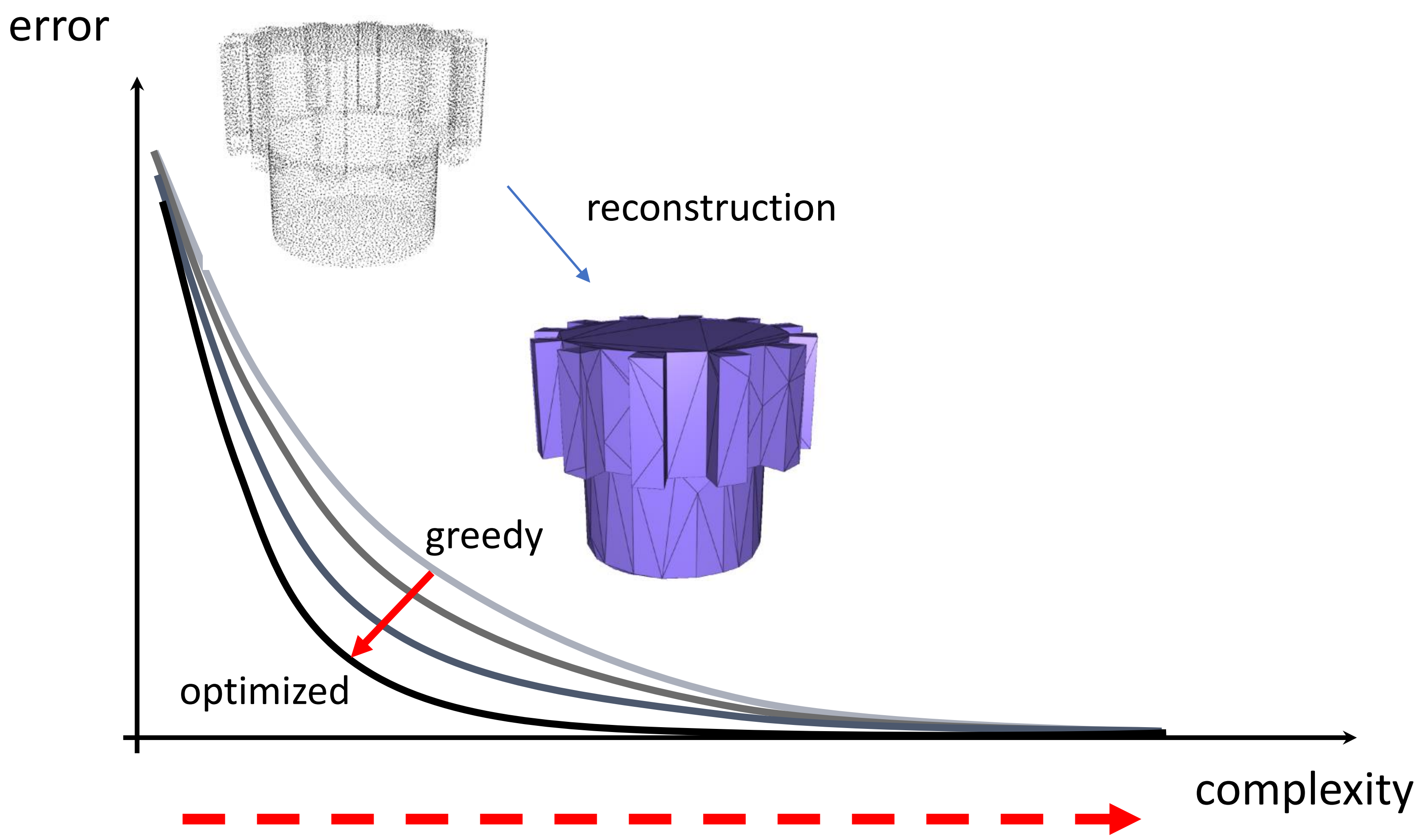


[Garland, Heckbert 97]

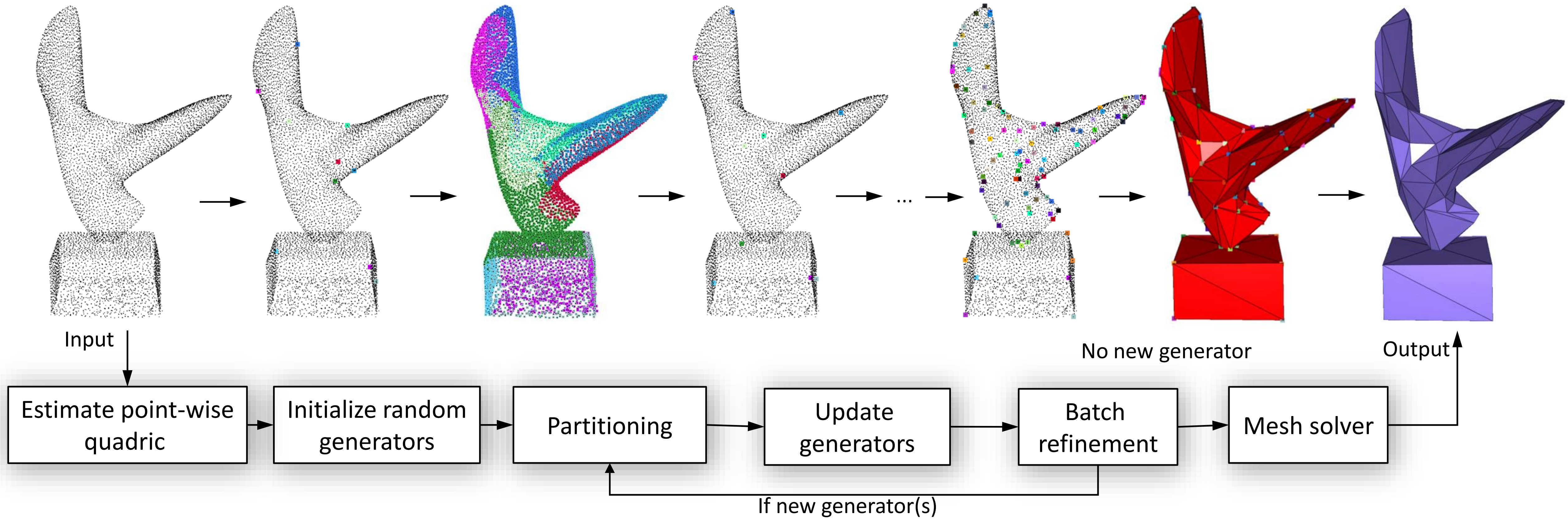
Complexity-Error Tradeoff



Objective



Pipeline



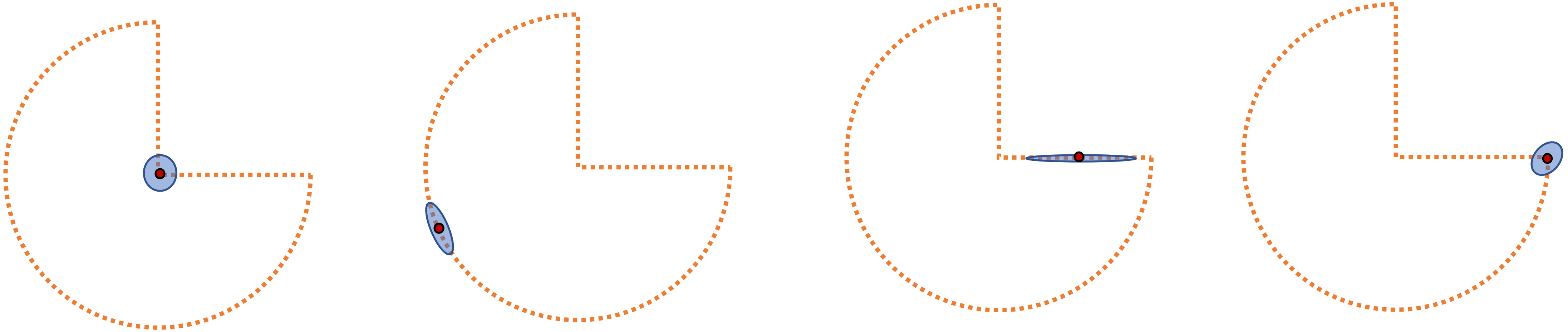
Input
...
No new generator
Output
Estimate point-wise quadric
Initialize random generators
Partitioning
Update generators
Batch refinement
Mesh solver
If new generator(s)

QEM Initialization

1. Point QEM: $Q_{p_i} = [n_i, p_i] \cdot [n_i, p_i]^T$

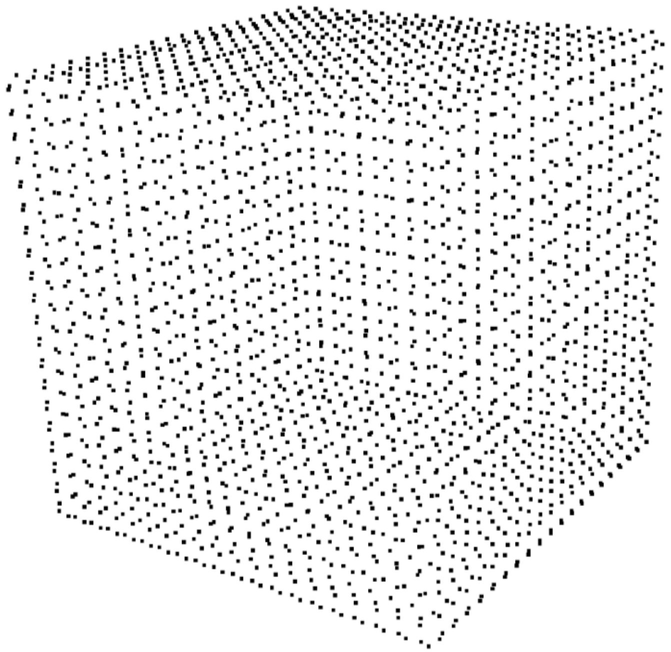
2. Point area: $a_{p_i} = \frac{1}{2k^2} \cdot \left(\sum_{p_j|(p_i,p_j)\in KNN(\mathcal{P})} \|p_i - p_j\|\right)^2$

3. Diffused QEM: $Q_{v_i} = \sum_{p_j|(p_i,p_j)\in KNN(\mathcal{P})} a_{p_j} \cdot Q_{p_j}$

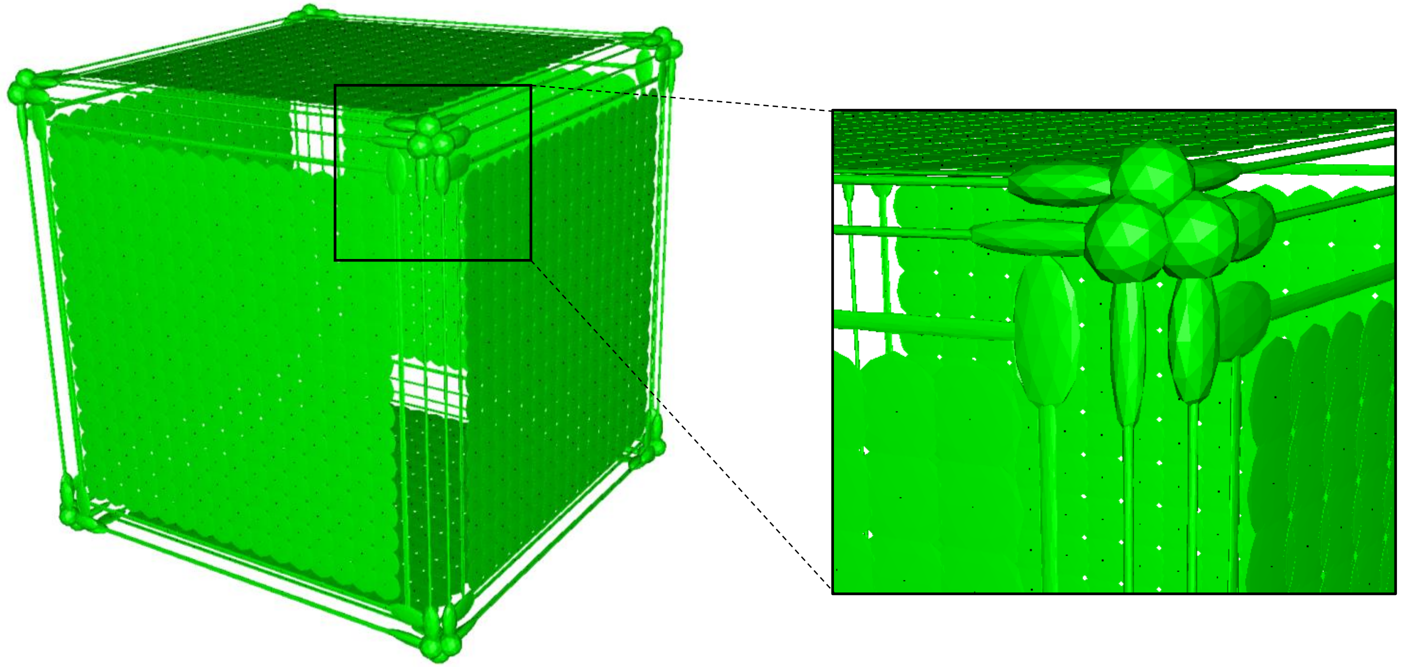


(a) Right corner: circle-like QEM (b) Smooth: ellipsoid-like QEM (c) Line: cigar-like QEM (d) Corner: ellipsoid-like QEM

QEM Initialization

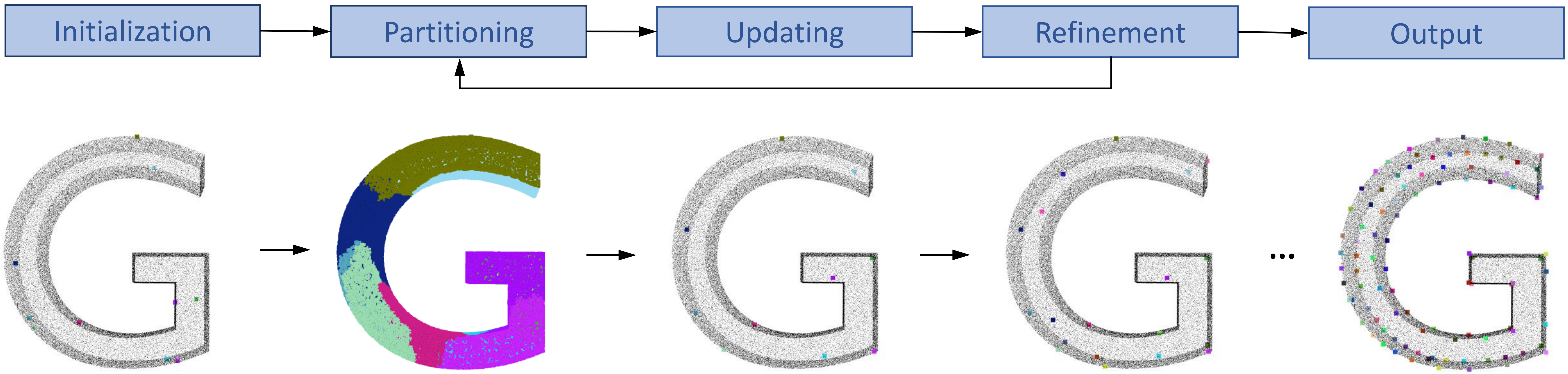


(a) Point cloud

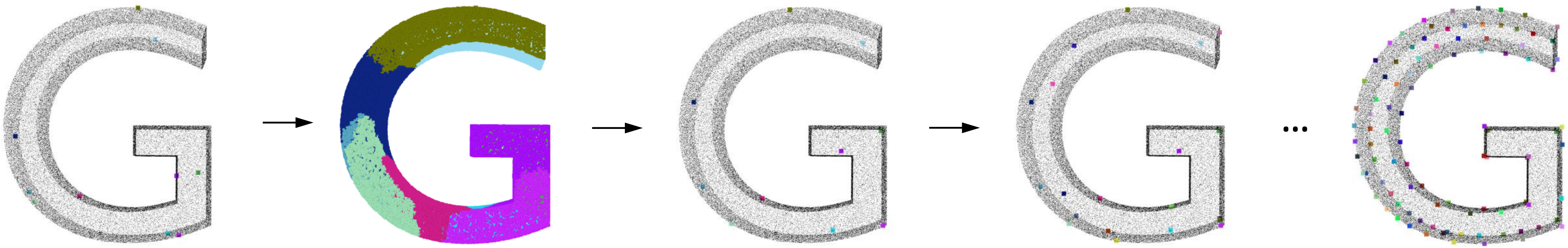


(a) Diffused QEM ellipsoids

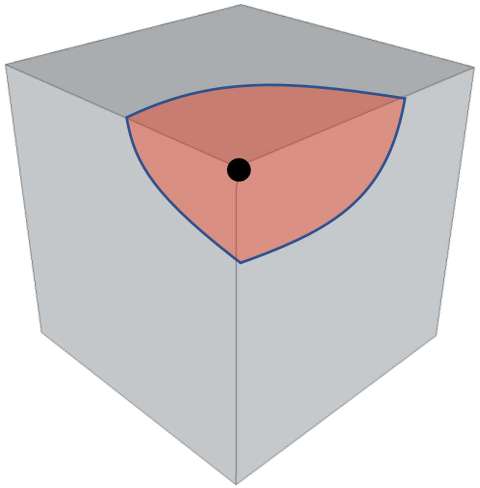
Clustering


Initialization
Partitioning
Updating
Refinement
Output
...

Clustering

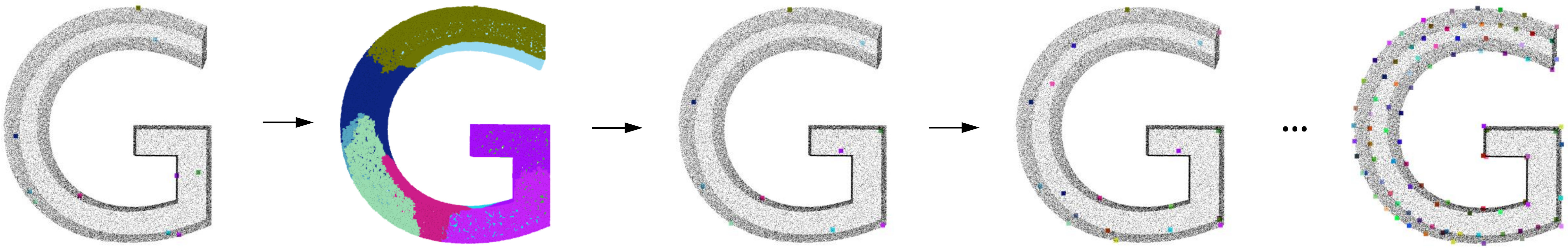


$$E\left(p_i, l_j\right) = \left[c_j, 1\right]^T \cdot Q_{v_i} \cdot \left[c_j, 1\right] + \lambda \cdot \left\|p_i - c_j\right\|^2$$



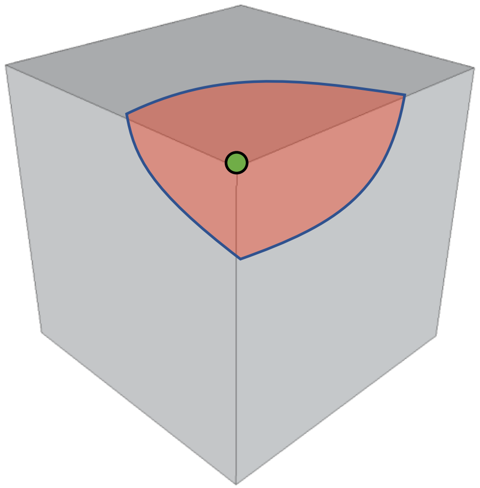
Clustering

Initialization → Partitioning → **Updating** → Batch splitting → Output

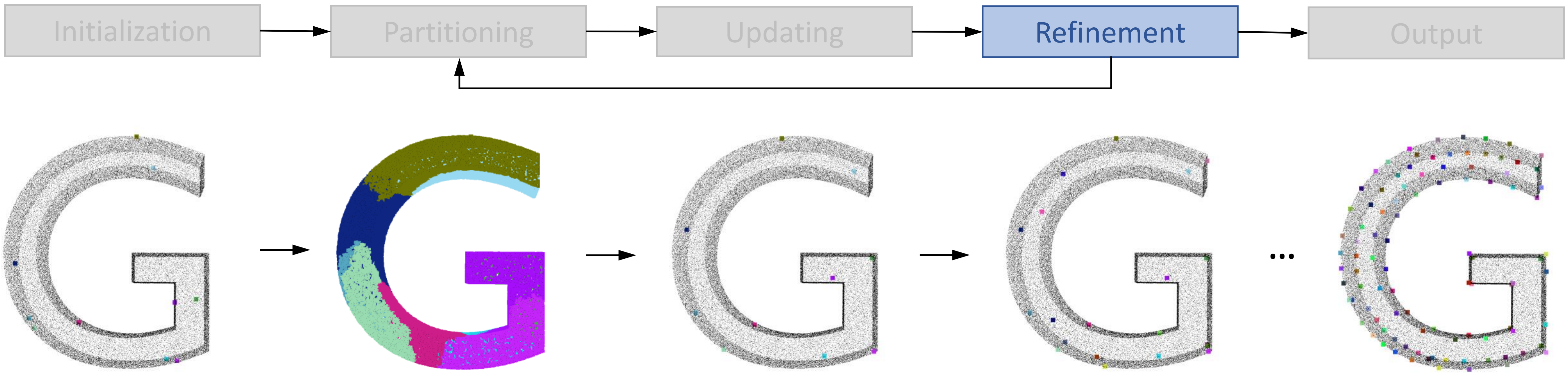


$$E(p_i, l_j) = [c_j, 1]^T \cdot Q_{v_i} \cdot [c_j, 1] + \lambda \cdot \|p_i - c_j\|^2$$

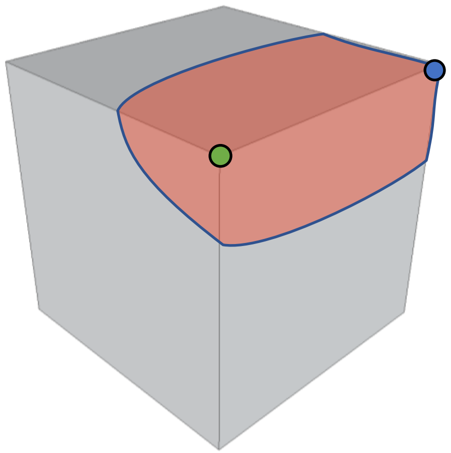
$$c_j^* = argmin_{p \in \mathbb{R}^3} [p, 1]^T \cdot Q_{c_j} \cdot [p, 1]$$



Clustering



$$E(p_i, l_j) = [c_j, 1]^T \cdot Q_{v_i} \cdot [c_j, 1] + \lambda \cdot \|p_i - c_j\|^2$$



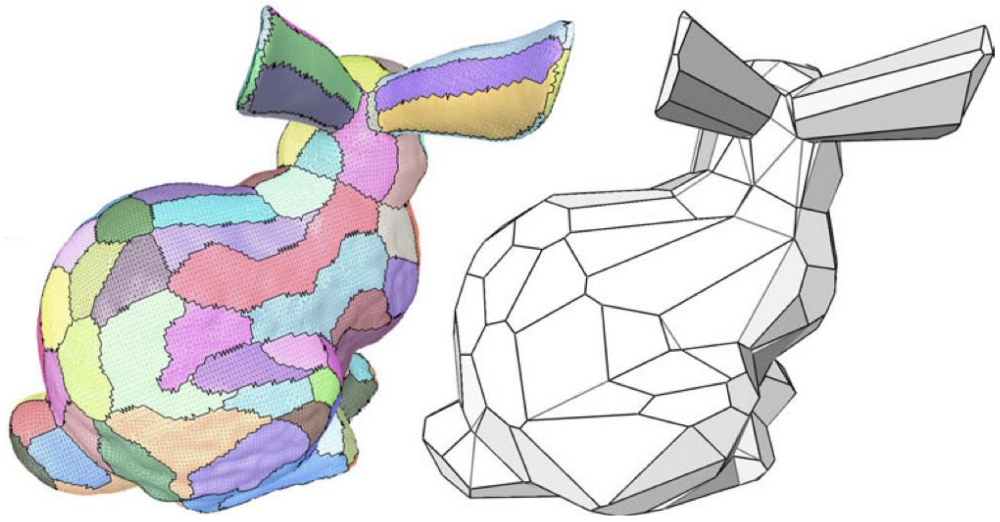
$$c_j^* = argmin_{p \in \mathbb{R}^3} [p, 1]^T \cdot Q_{c_j} \cdot [p, 1]$$

$$p_{max}(l_j) = argmax_{p_i \in l_j} [p_i, 1]^T \cdot Q_{c_j} \cdot [p_i, 1]$$

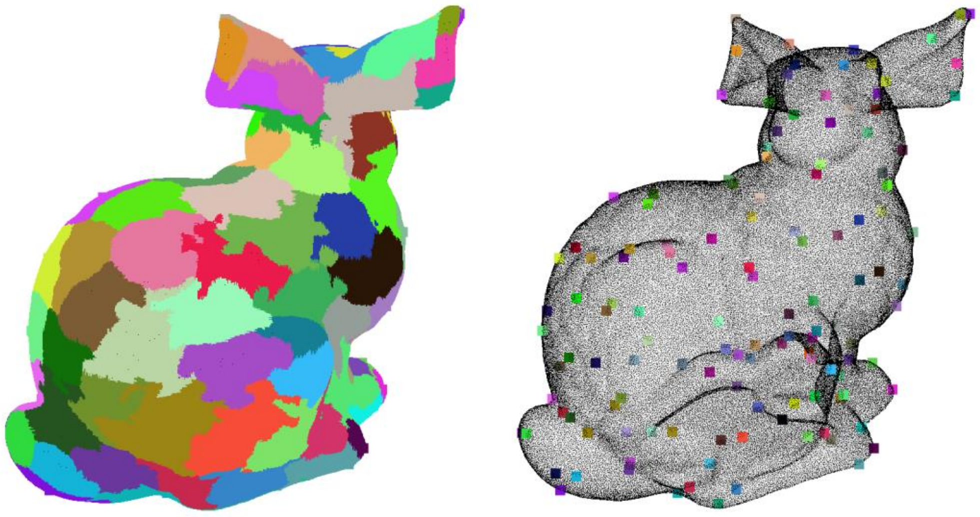
Meshing

Variational shape approximation[1] : “dual“ partitioning $\longrightarrow$ not trivial to extract triangle mesh

Our approach: optimized generators $\longrightarrow$ vertices of the output triangle mesh



VSA[1]



This approach

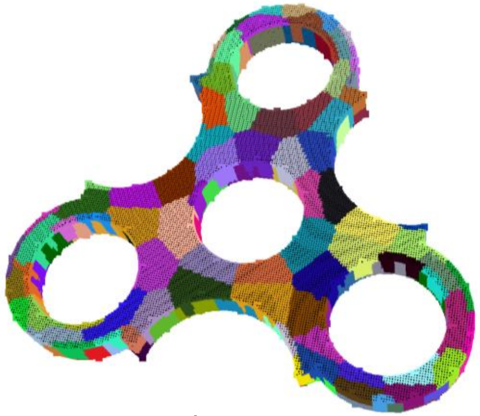
[1] Cohen-Steiner et al. Variational Shape Approximation. ACM SIGGRAPH, 2004.

Meshing

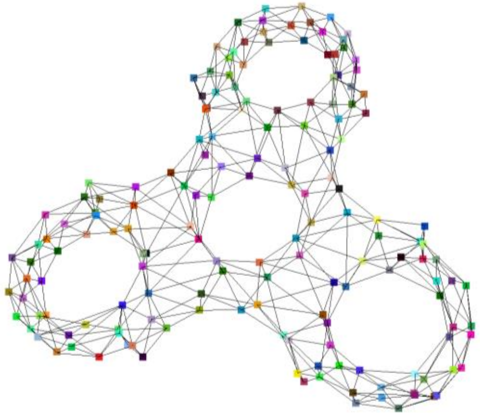
1. Construct edge candidate set: connect adjacent clusters

2. Construct facet candidate set: find 3-cycles

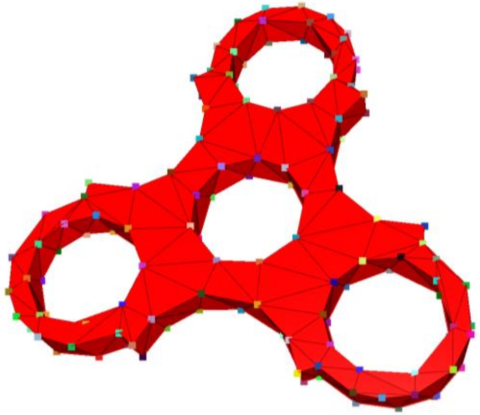
3. Mesh extraction via Binary Integer Programming (BIP) solver [1]



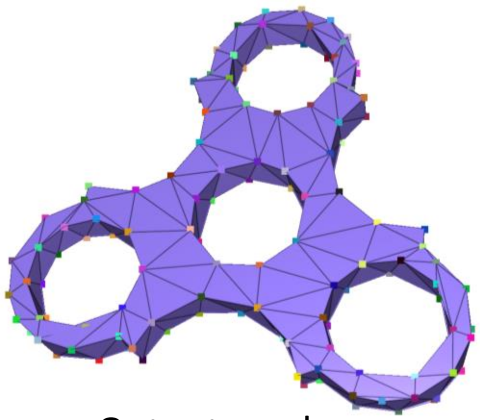
Clusters



Edge graph



Facet candidates



Output mesh

[1] Nan, Liangliang, et al. Polyfit: Polygonal surface reconstruction from point clouds. Proceedings of the IEEE International Conference on Computer Vision. 2017.

Meshing

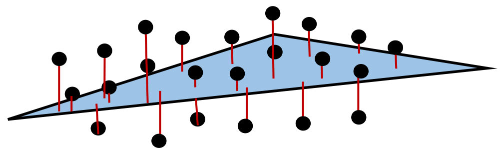
Fitting term

Coverage term

Manifold constraint

$$E(\mathcal{T}) \;=\; \lambda_f \sum_{i=1}^{n} b_{f_i} \cdot F_f(f_i)$$

$$F_f(f_i) = \sum_{p_j | d(p_j, f_i) < \epsilon} \left(1 - \frac{d(p_j, f_i)}{\epsilon}\right)$$

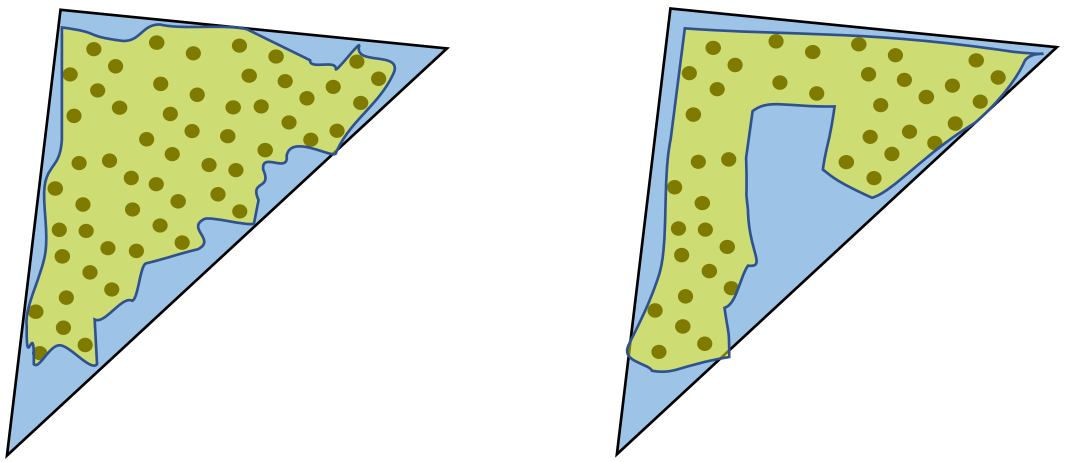


Meshing

Fitting term | **Coverage term** | Manifold constraint

$$E(\mathcal{T}) = \lambda_f \sum_{i=1}^{n} b_{f_i} \cdot F_f(f_i) + \lambda_c \sum_{i=1}^{n} b_{f_i} \cdot F_c(f_i)$$

$$F_c(f_i) = min\left(1, \frac{area(\alpha(\{p_j | d(p_j, f_i) < \epsilon\}))}{area(f_i)}\right)$$



$F_c(f_i) = 0.95$ $\qquad$ $F_c(f_i) = 0.65$

Meshing

Fitting term

Coverage term

Manifold constraint

$$E(\mathcal{T}) \;=\; \lambda_f \sum_{i=1}^{n} b_{f_i} \cdot F_f(f_i) \;+\; \lambda_c \sum_{i=1}^{n} b_{f_i} \cdot F_c(f_i) \qquad s.t. \quad 2b_{e_i} - \sum_{f_j\ around\ e_i} b_{f_j} = 0, \qquad \forall e_i$$

If $b_{e_i} = 1$, 2 faces around e_i are selected

If $b_{e_i} = 0$, no face around e_i is selected

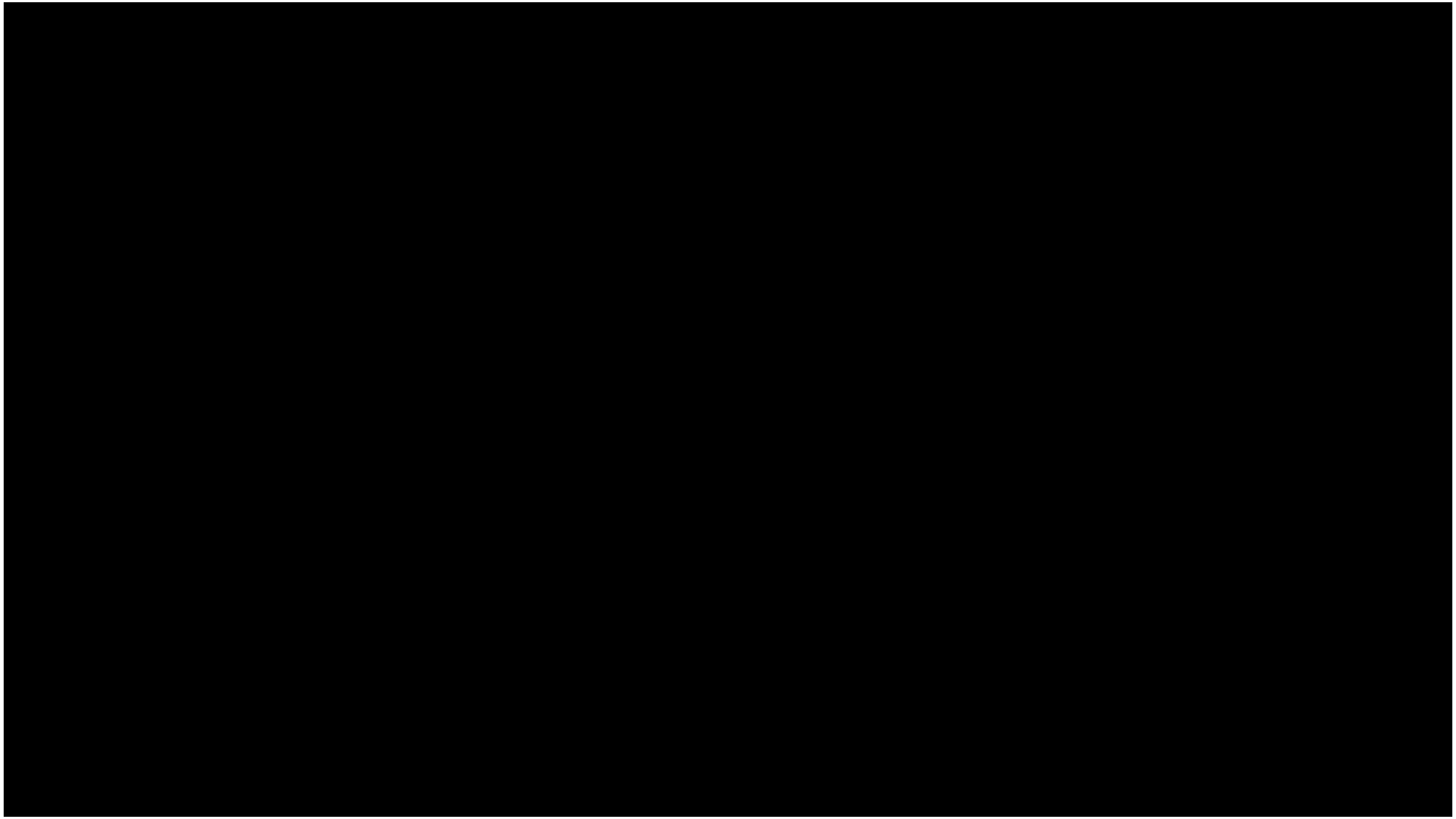
$\downarrow$

Manifoldness

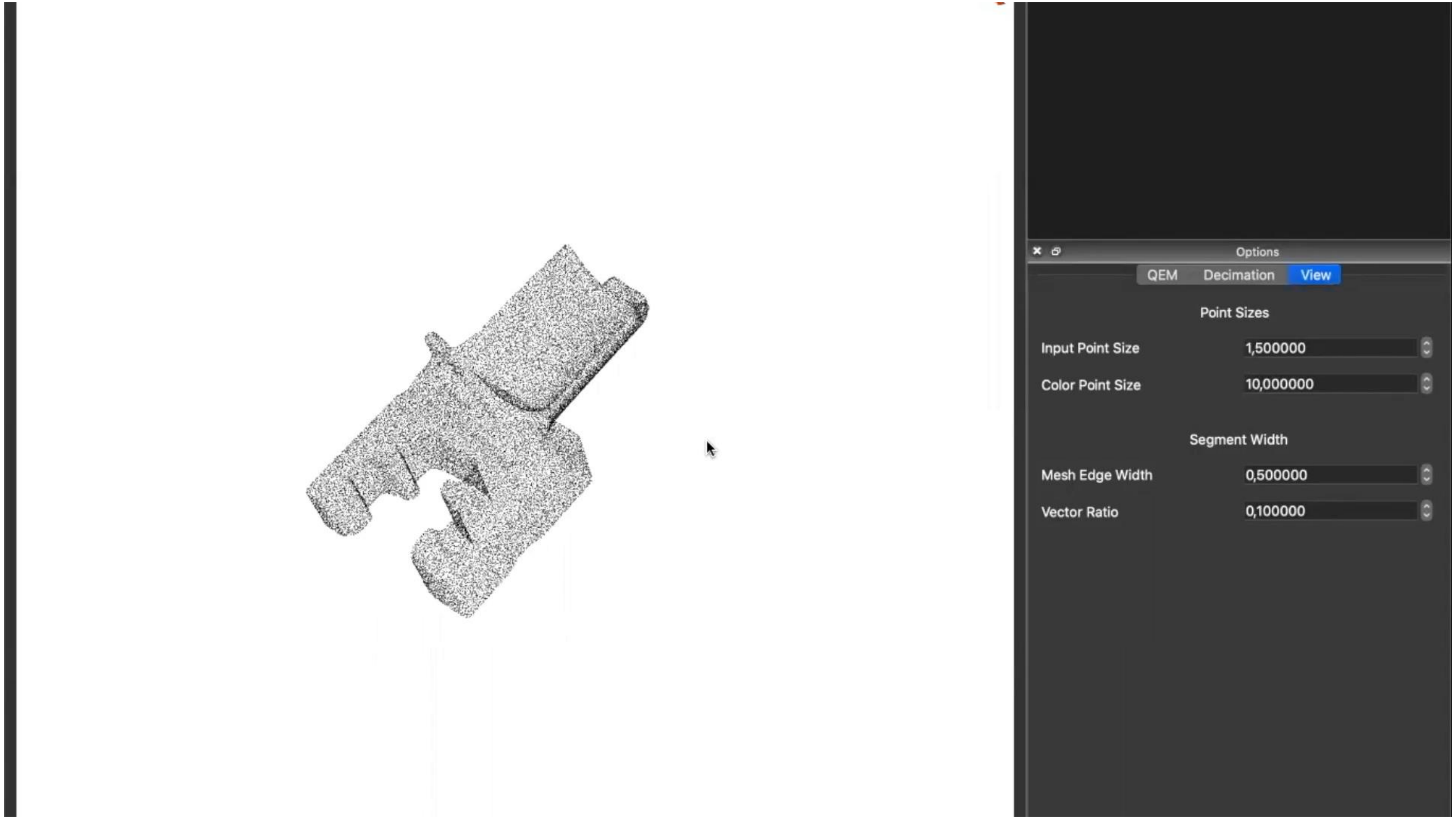
Implementation

- CGAL
- Eigen (quadrics)
- Solver: SCIP (mixed integer programming)

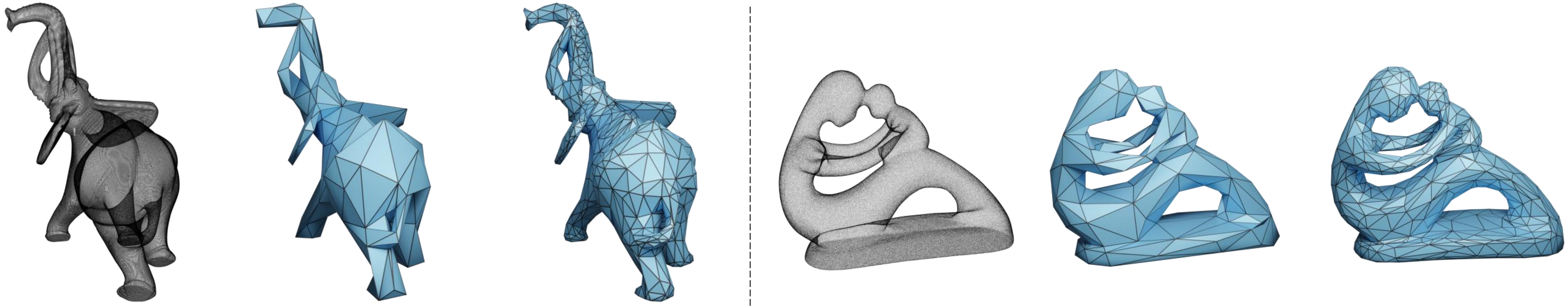
Hand: A Free-form Model



Blade: A Piecewise-smooth Model



Qualitative Results



#p: 1,537,296

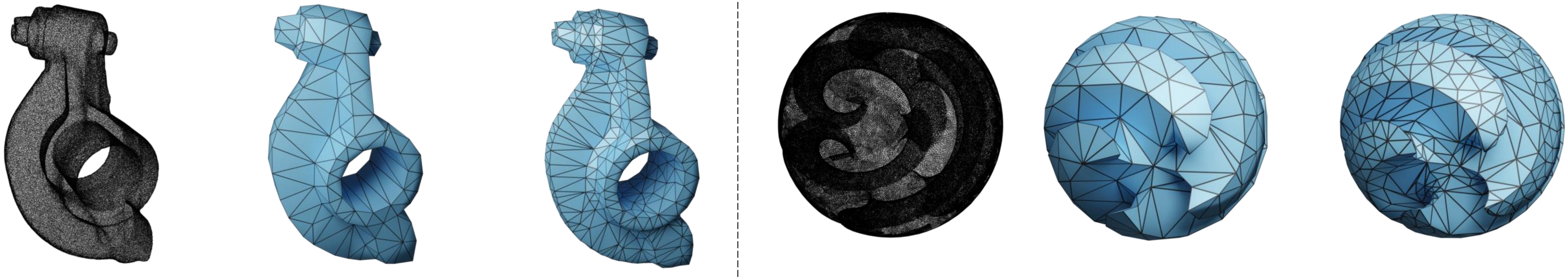
#v: 148
#f: 300

#v: 813
#f: 1,626

#p: 291,569

#v: 159
#f: 330

#v: 638
#f: 1,284



#p: 145,617

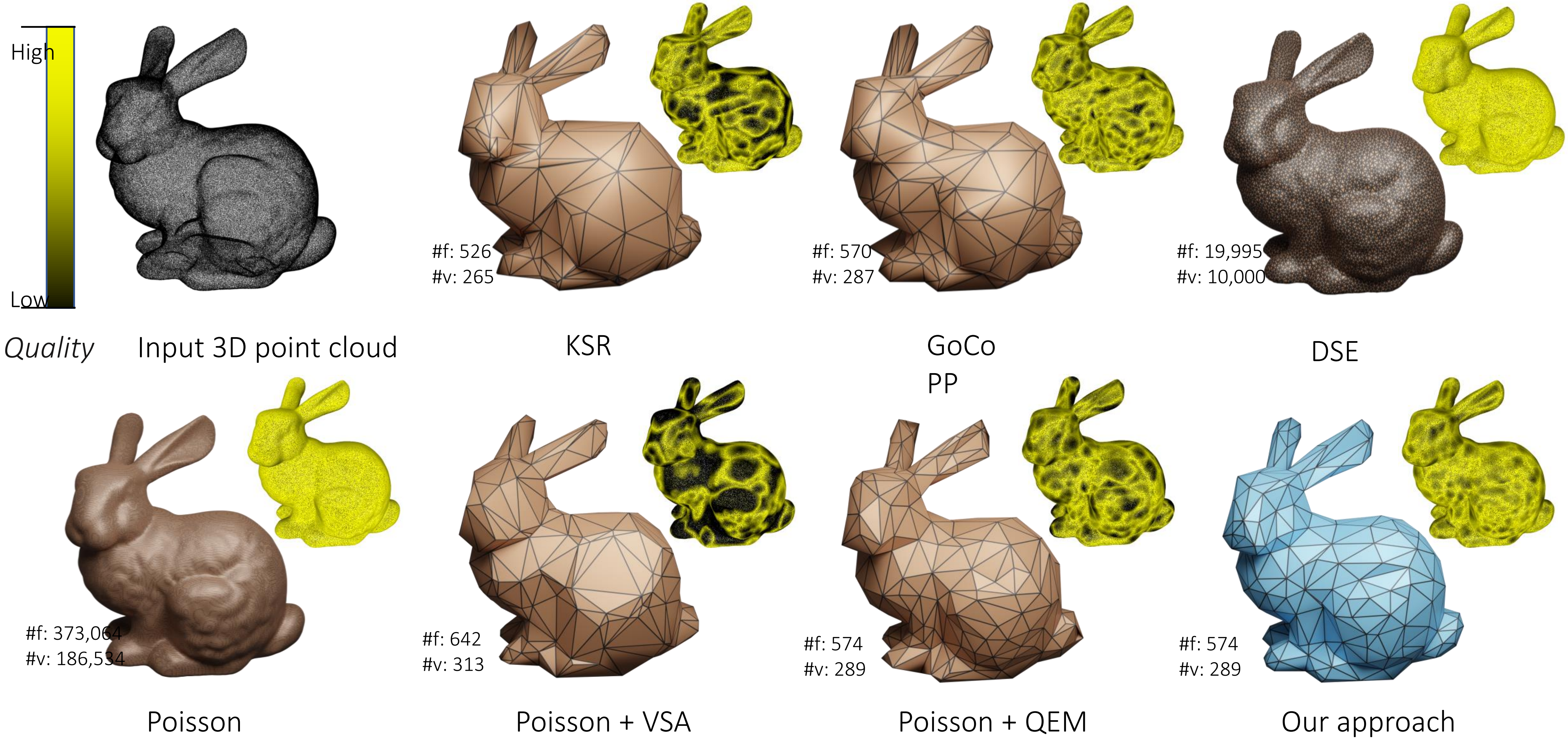
#v: 200
#f: 400

#v: 581
#f: 1,162

#p: 286,945

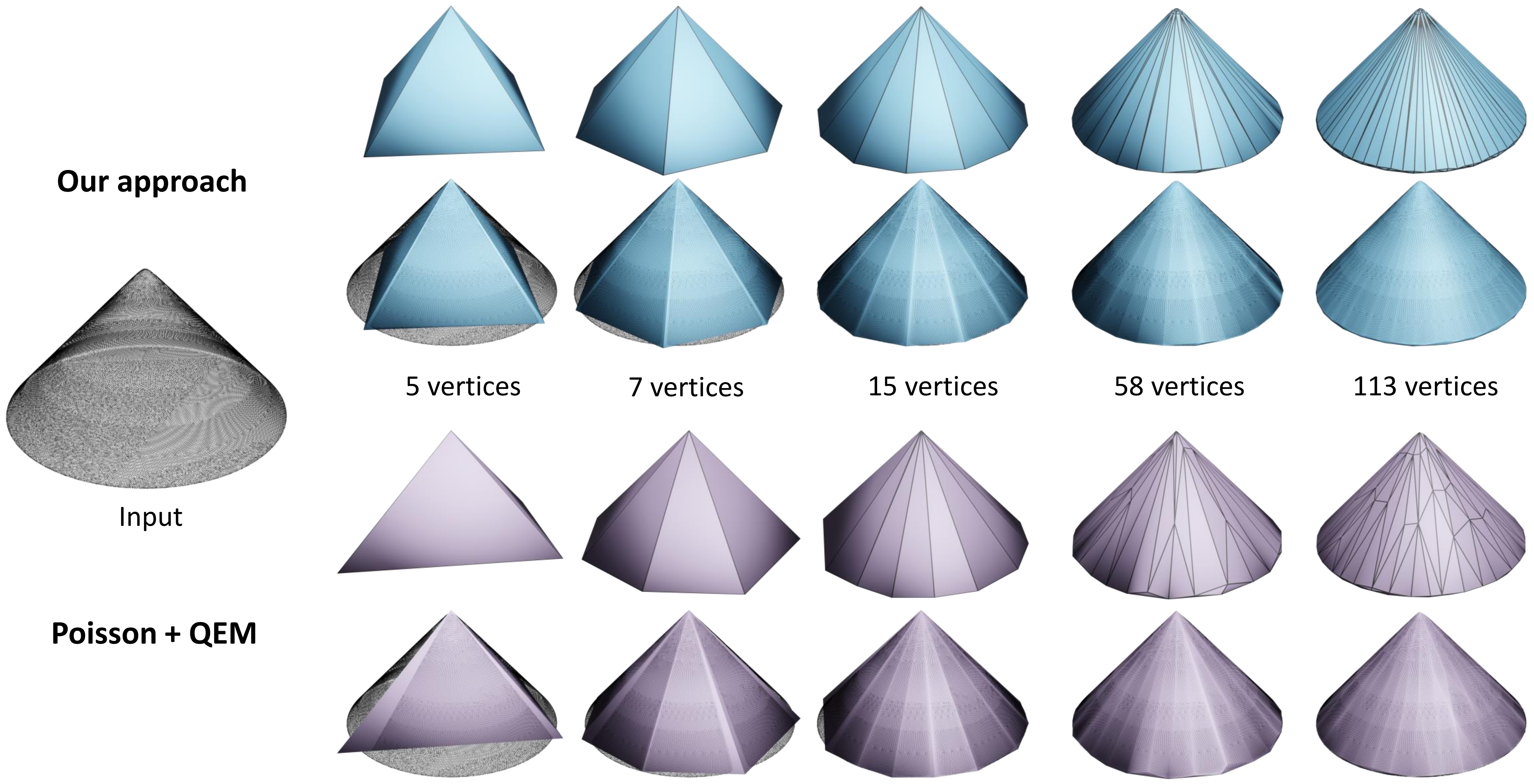
#v: 301
#f: 598

#v: 975
#f: 1,946

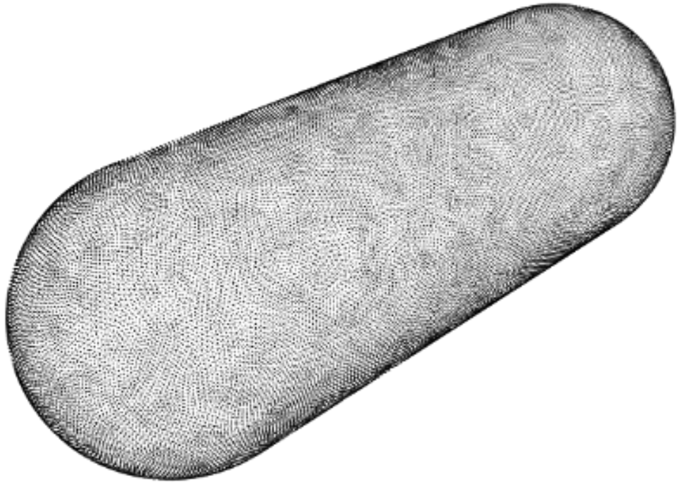


[1] Bauchet, Jean-Philippe, et al. Kinetic shape reconstruction. ACM TOG 39(5), 2020.
[2] Yu, Mulin, et al. Finding Good Configurations of Planar Primitives in Unorganized Point Clouds. IEEE CVPR, 2022.
[3] Rakotosaona, Marie-Julie, et al. Learning delaunay surface elements for mesh reconstruction. IEEE CVPR, 2021.
[4] Kazhdan, Michael, et al. Screened poisson surface reconstruction. ACM Transactions on Graphics 32, no. 3, 2013.
[5] Cohen-Steiner, David, et al. Variational shape approximation. ACM SIGGRAPH, 2004.
[6] Garland, Michael, and et al. Surface simplification using quadric error metrics. ACM SIGGRAPH. 1997.

Variational VS Greedy

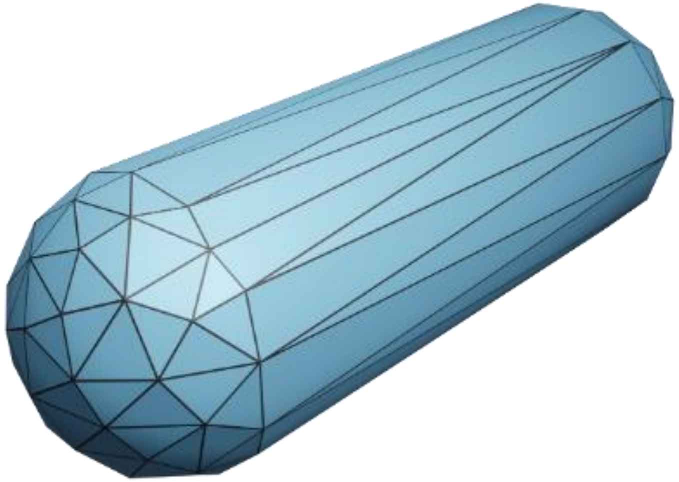


Capsule



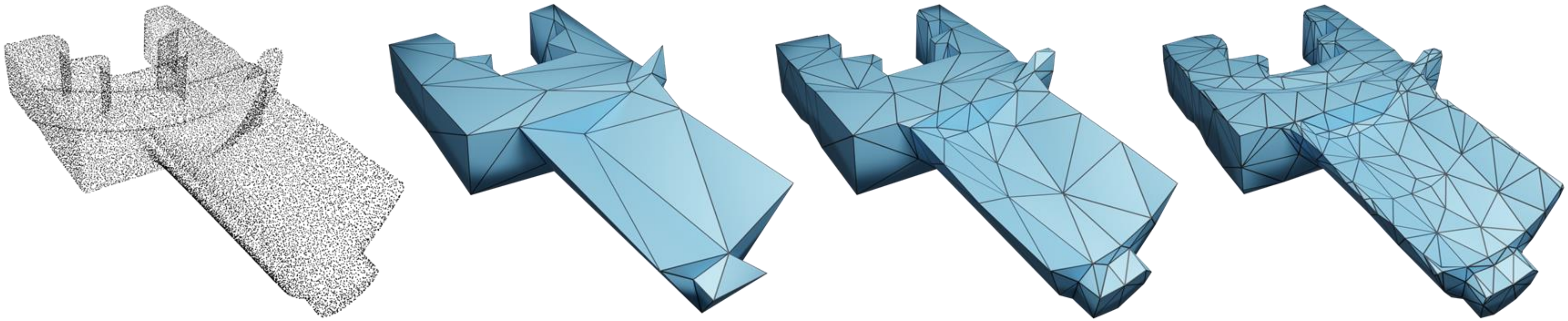
Input 3D point cloud

Poisson + QEM



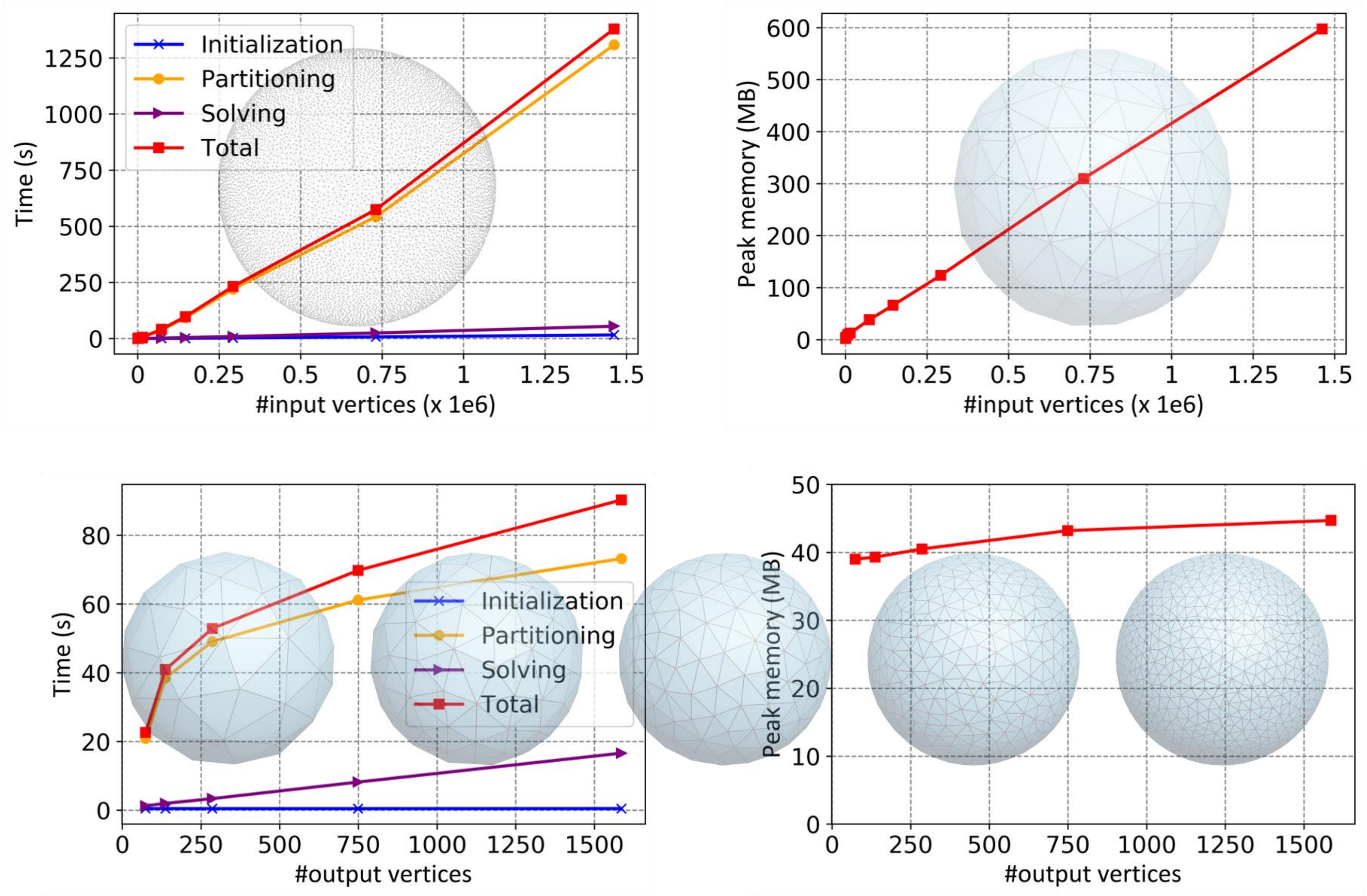
Our approach

Blade

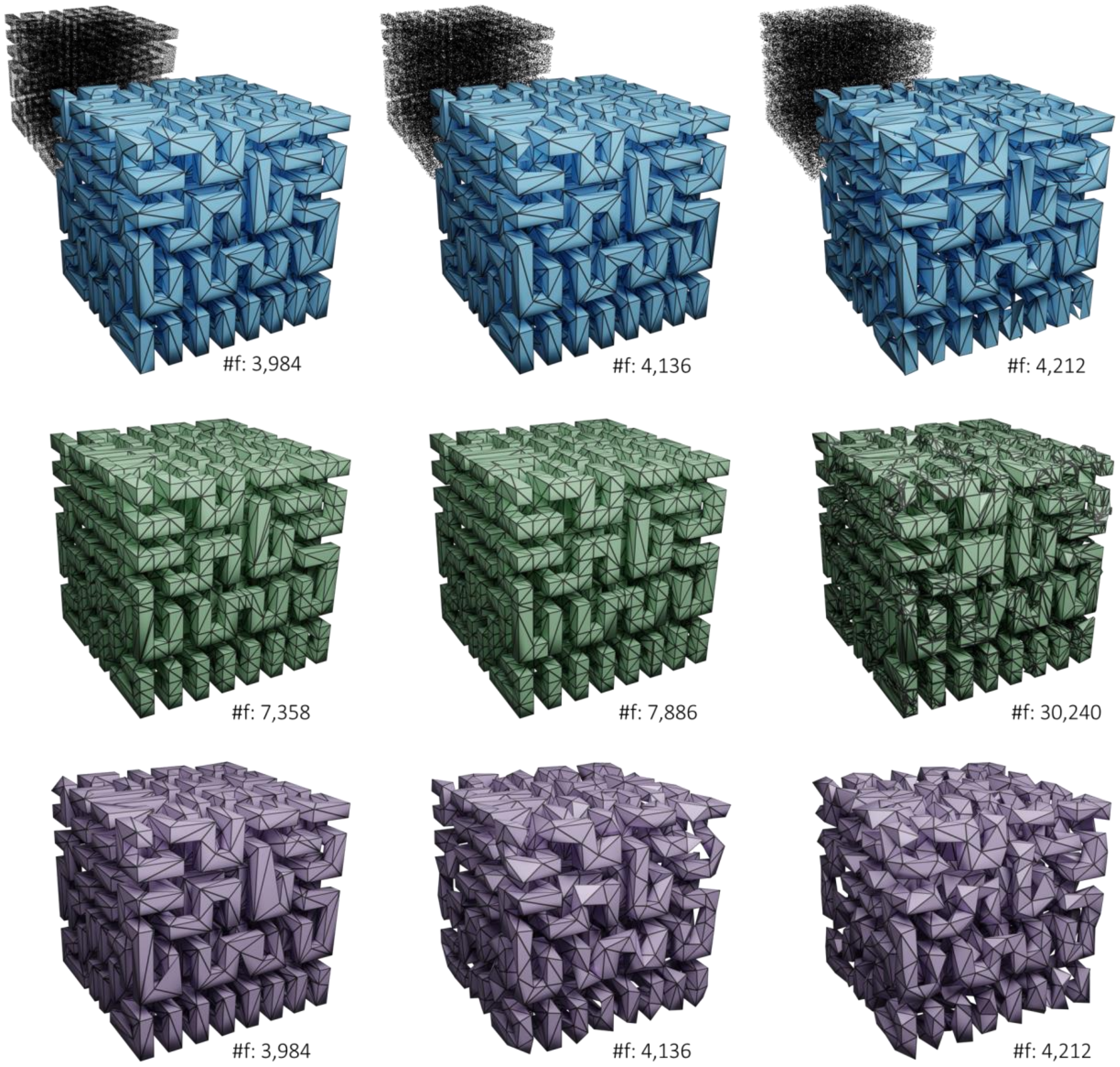


Input 3D point cloud   Coarse   Medium   Dense

Timing and memory

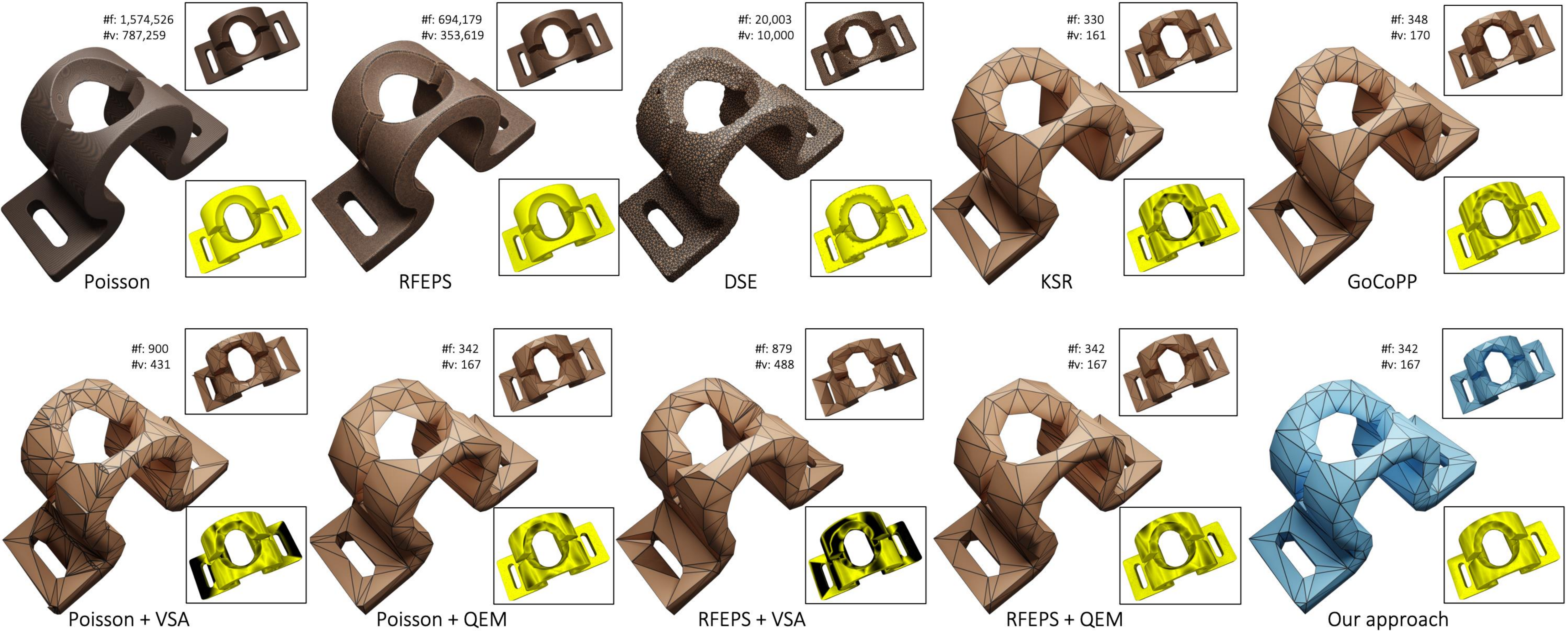


Robustness to Noise



No noise 0.5% noise 1% noise

Result


#f: 1,574,526
#v: 787,259
Poisson
#f: 694,179
#v: 353,619
RFEPS
#f: 20,003
#v: 10,000
DSE
#f: 330
#v: 161
KSR
#f: 348
#v: 170
GoCoPP
#f: 900
#v: 431
Poisson + VSA
#f: 342
#v: 167
Poisson + QEM
#f: 879
#v: 488
RFEPS + VSA
#f: 342
#v: 167
RFEPS + QEM
#f: 342
#v: 167
Our approach

One-sided Hausdorff Errors

Model	Capsule	Cone (#v:5)	Cone (#v:7)	Cone (#v:15)	Cone (#v:58)	Cone (#v:113)	Hilbert	Hilbert (0.5%)	Hilbert (1%)
Max (P+Q)	0.07489	0.3836	0.1078	0.06092	0.01455	0.008757	1.6725	1.7233	1.5873
Max (Ours)	0.03256	0.2561	0.08905	0.01721	0.007488	0.004118	0.1729	1.0565	2.0487

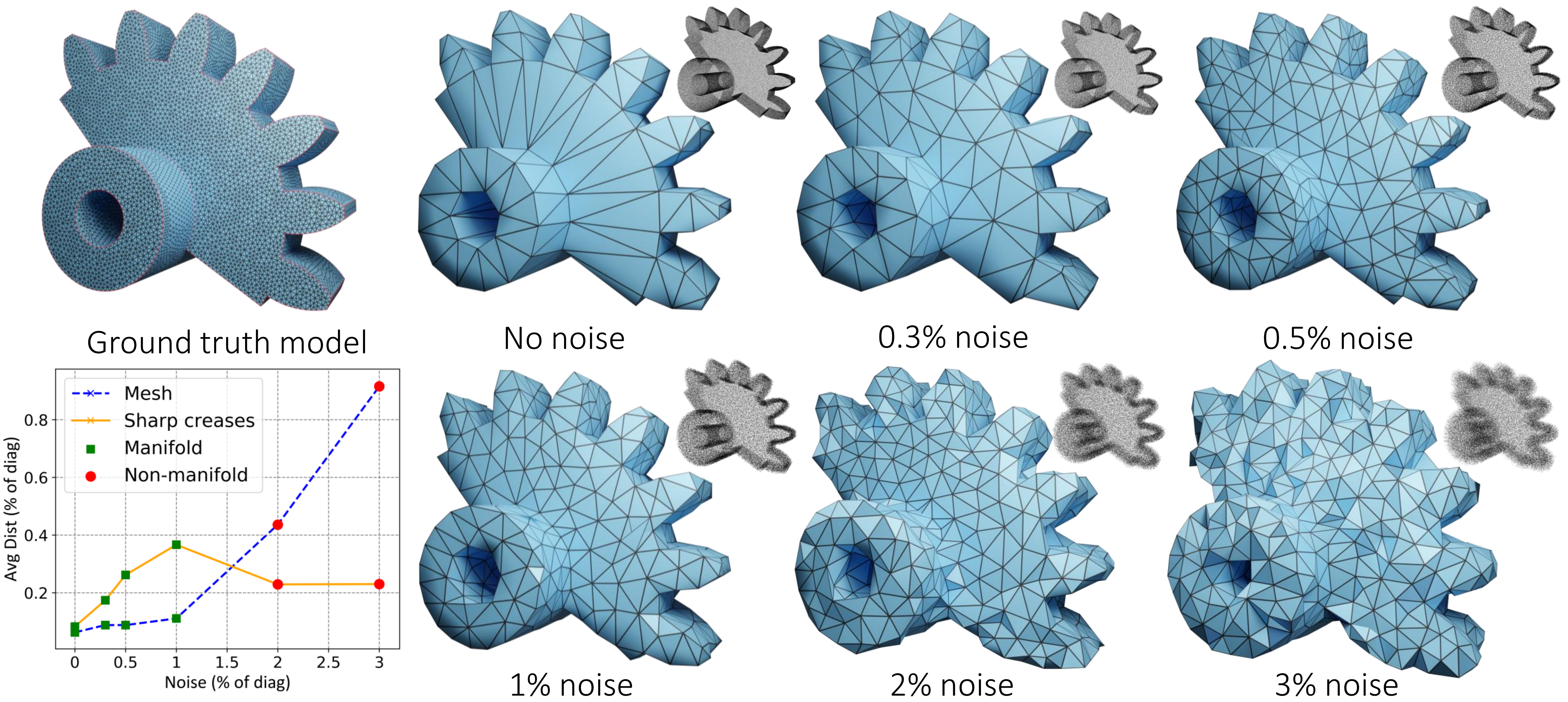
Model	Bunny	Part	Hand	Elephant (Middle)	Elephant (Right)	Mother (Middle)	Mother (Right)	Rocker arm (Middle)	Rocker arm (Right)
Max (P+Q)	0.00328	2.145	0.01345	0.04299	0.01088	4.8591	1.3958	0.002321	0.001147
Max (Ours)	0.002055	2.1063	0.008587	0.0232	0.007387	4.5793	1.1293	0.001209	0.0005845

Table 1. Maximum distances from 3D point clouds to output meshes.

On-going work

- Initialization
  - One quadric per input point vs field of quadric ?
  - Learning to regress quadrics ?
  - Boundaries & sharp features ?
- Surfaces with boundaries
- 2-Manifold: hard constraint vs objective function
- Valid mesh as output? (no self-intersections)
- Strict error tolerance: 1-sided, 2-sided Hausdorff

Thank you.


Ground truth model
No noise
0.3% noise
0.5% noise
1% noise
2% noise
3% noise
Mesh
Sharp creases
Manifold
Non-manifold
Avg Dist (% of diag)
Noise (% of diag)
0.2
0.4
0.6
0.8
0
0.5
1
1.5
2
2.5
3

Summary

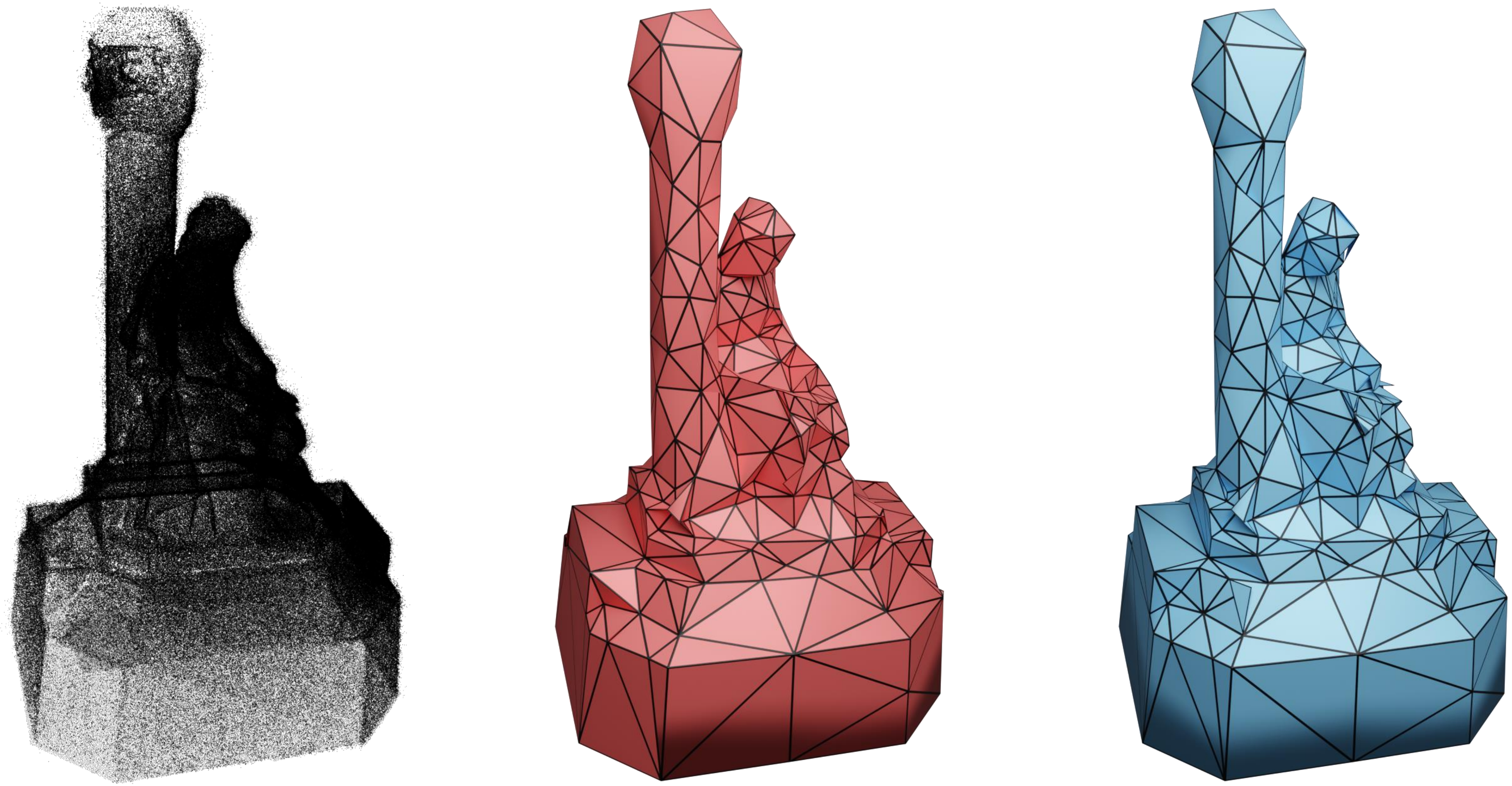
Variational shape reconstruction via quadric error metrics

Prior(s)

- Piecewise-smooth prior

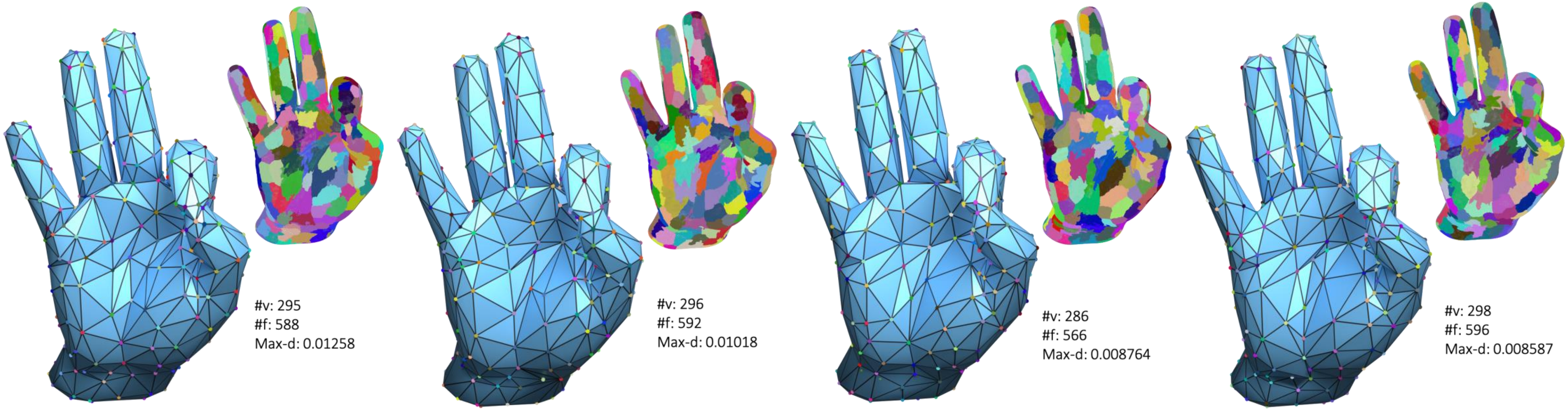
Properties

- Input/output-aware
- Progressive
- Error tolerance
- Feature-preserving
- Concise

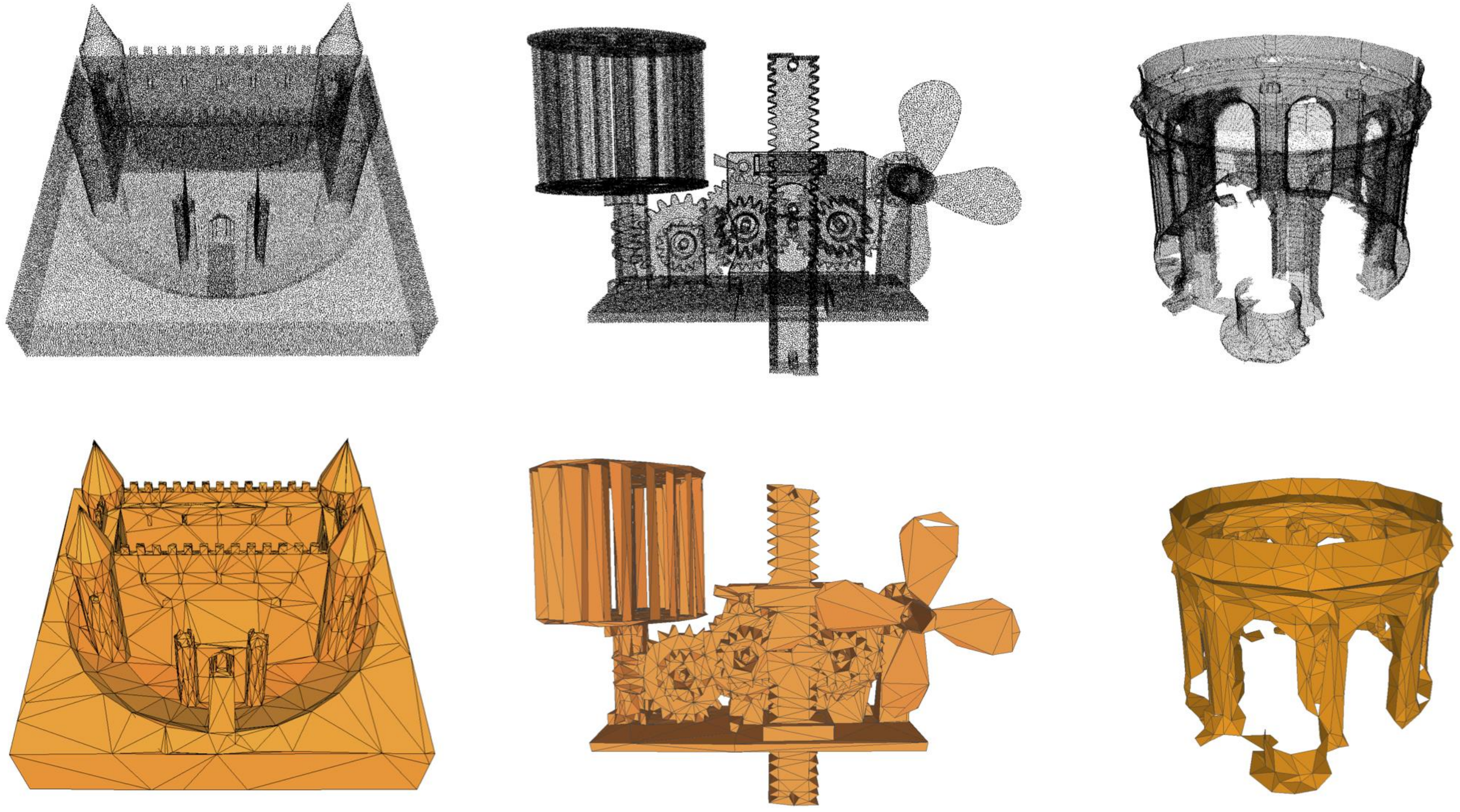


Noisy point cloud　　Face candidates　　Reconstructed mesh

Initialization



Non-manifold solver



Castle Full thing Saint-Jean